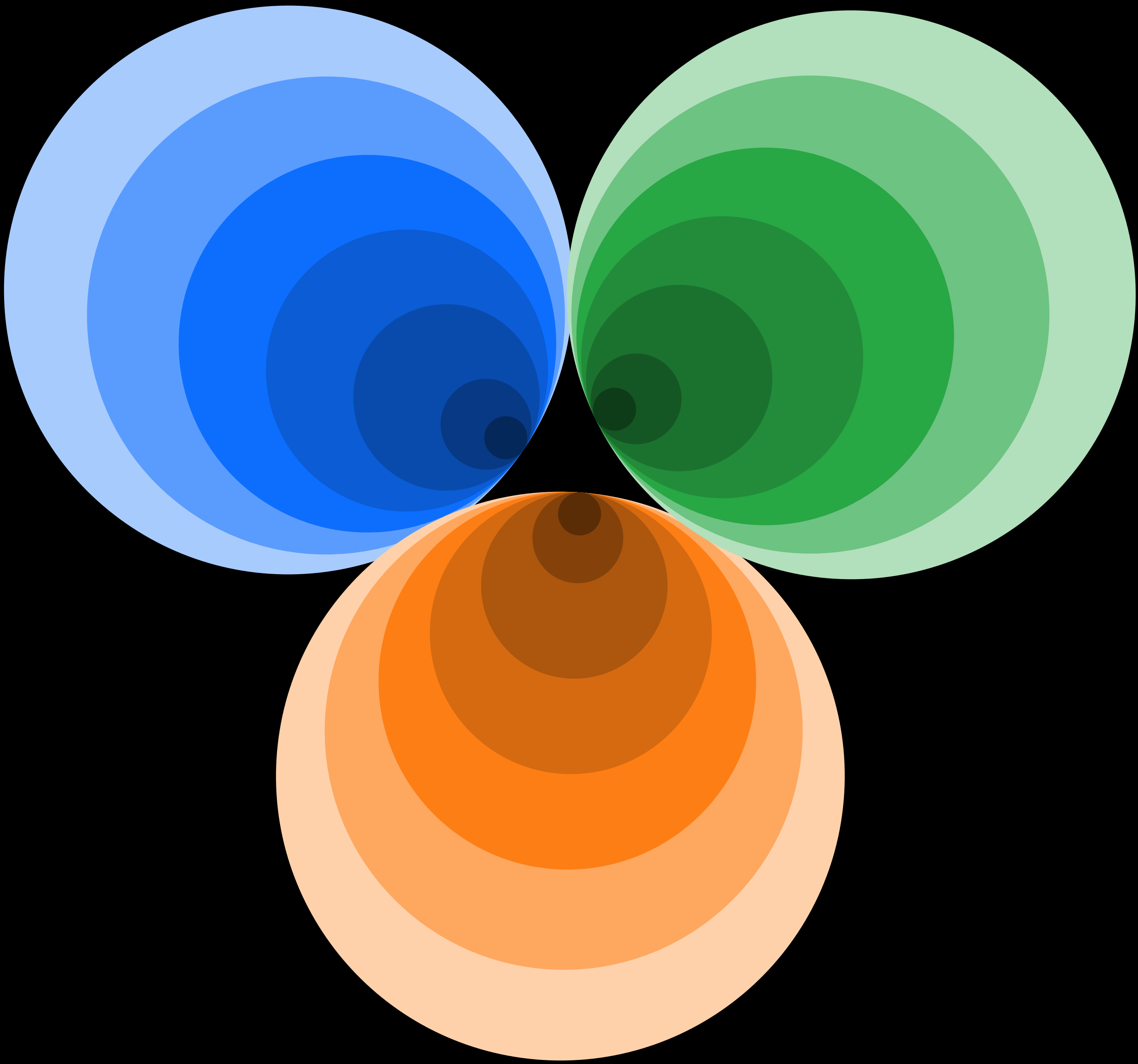
Ya. B. Zel'dovich

THEORY OF SHOCK WAVES AND INTRODUCTION TO GAS DYNAMICS



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English pages: 231

Translated under: Contract AF33(657)-14673, SA1

Akademiya Nauk Soyuza SSR

Institut Khimicheskoy Fiziki

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TEORIYA UDARNYKH VOLN I VVEDENIYE V GAZODINAMIKU

Izdatel'stvo Akademii Nauk SSSR

Moskva 1946 Leningrad

186 pages

Gas dynamics is defined as the science of motion at great pressure differentials and high velocities, velocity being measured in terms of the speed of sound. The book confines itself to specific phenomena of gas dynamics, i.e., those which have no analogies in the mechanics of an incompressible liquid. Emphasis is placed on careful definition of the fundamentals of gas dynamics, fundamental laws, and methods of solving simplest problems, rather than on the computational methods of gas dynamics or methods of numerical integration of complex two- and three-dimensional flows, etc. Attention is devoted to problems of flow around bodies moving at great speeds, motion of a gas in ducts such as nozzles and pipes, and compressibility of the moving medium. The second main topic is shock waves considered under the theory of shock waves, shock wave laws, and the problem of the destructive effect of explosions and propagation of the explosion on the explosive substance (capable of chemical reaction). The author intends the text to also serve as an introduction to the theory of explosions.
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Introduction

Gas dynamics is a component part of hydrodynamics, the science of fluids, liquids and gases.

A particular feature of gas dynamics is the need to keep account of the compressibility of the medium. Liquids may be considered incompressible under normal circumstances, whereas gases change their volume considerably even under a slight variation in pressure.

It is obvious that specific formulas and laws of gas dynamics have to be applied to gases only insofar as we are dealing with pressure changes of great magnitude.

In the case of small velocities, the motion of gas can be regarded in the same way as the motion of a liquid, i.e., ignoring the change of volume and compressibility.

Depending upon the condition, the order of magnitude of pressure differentials arising in a flow changes from $\rho u^2/2$ the value of dynamic impact according to Bernoulli's formula, to ρuc, where c is the speed of sound, u is the speed of motion and ρ is gas density. Gas pressure is approximately equal to ρc^2.

If we juxtapose the expressions, we see that at subsonic velocities the pressure differentials are small as compared with pressure proper and, consequently, we may therefore, as a rule, ignore the compressibility of the medium.

Following is a definition of the scope of gas dynamics. Gas dynamics is the science of motion at great pressure differentials and high velocities, velocity being measured in terms of the speed of sound.

In similarity theory we have the following ratio between motion and speed of sound:

$$u/c = Ba$$

where Ba is known as the Barstow criterion.

Gas dynamics studies motion and Ba values close to unity. If Ba is considerably smaller than 1, the general equation of gas dynamics becomes those of hydrodynamics of an incompressible liquid.

It will be assumed in the following that laws of hydrodynamics of an incompressible liquid are known, and we shall therefore not dwell on the derivation of the corresponding formulas.

To take account of compressibility means that one also has to take account of the change in the state of the medium. In hydrodynamics the action of dissipative forces (viscosity) leads to a release of heat in the liquid and to a change in its temperature, but it does not lead to a change in volume: the changes within the liquid have no inverse effect on the nature of the flow and have little importance for the phenomena investigated in hydrodynamics.

In gas dynamics, instead, we shall continuously deal with changes in the state of the medium in the flow proper. This aspect of gas dynamics requires that any and all phenomena be also investigated from a thermal dynamic point of view; thus, thermodynamics is totally indispensable for the study of gas dynamics

In the present book we shall deal only with specific phenomena of gas dynamics, i.e., such that have no analogies in the mechanics of an incompressible liquid. We shall not dwell on those subjects in which gas dynamics and the consideration of compressibility give only slight correction for the conventional formulas of hydrodynamics of an incompressible liquid. The emphasis in the present book will be on the careful definition of the fundamentals of gas dynamics, of the fundamental laws, and of the methods for solving the simplest problems, rather than on the computational methods of gas dynamics, the methods of numerical integration of complex two- and three-dimensional flows, etc. We shall proceed here from the simple to the complex, rather than from general problems to particular ones. Instead of writing first the equations of gas dynamics in their most general form (taking into consideration all the factors), searching for general solutions and then, by simplifying these solutions, going on to the particular solution of simple cases, we shall solve simple, elementary problems that describe certain aspects of some phenomena, and then, by means of these individual partial solutions piece together the solution of more complex problems.

We can outline the following, fundamental fields of application of gas dynamics. The first, which today is the better known and more developed one, comprises problems of flow around bodies moving at great speeds. This involves, first of all, the corrections in ordinary formulas of resistance and lift for bodies moving at subsonic speeds, i.e., corrections that are already applicable to contemporary aviation. A radical change in flow around bodies occurs when we deal with velocities exceeding the speed of sound. These speeds are involved in ballistics, i.e., the science of the motion of missiles and projectiles, and also in the study of rocket aircraft of the near future.

This application of gas dynamics to the problem of the motion of a body in a gas at speeds of the order of the speed of sound or exceeding it is dealt with in detail in text books, hence we shall deal with it only marginally here.

The second, extremely important field is that of the motion of a gas in ducts, such as nozzles and pipes. Again, gas dynamics becomes indispensable if and when the velocity of the gas attains or exceeds the speed of sound. In this field, the nature of the flow, and the dependence of velocity and flow rate on pressure drop, are subject to qualitative changes. This group of problems is of great significance for the theory of turbines, jet engines and missiles.

A peculiar field of gas dynamics based on the consideration of the compressibility of the moving medium is the teaching on sound — acoustics. The velocity of the medium and the amplitude of pressure changes under the effect of sound are very small. Nevertheless, consideration of compressibility becomes indispensable when studying the initial stages of any motion, and when studying rapidly changing, especially periodical motion.

Shock waves are of particular interest from various points of view, and they will be one of the main subjects of the present book. On the one hand, wherever the attempts of integrating equations without introducing discontinuities (i. e., shock waves) lead to paradoxes which make it impossible to solve these equations, the theory of shock waves eliminates the paradoxes and makes it possible to design a regime of motion under any conditions.

On the other hand, the shock waves themselves are a paradoxical phenomenon. They are paradoxical in that, without introducing any assumptions regarding dissipative forces (viscosity and thermal conductivity), from elementary considerations we can derive shock wave laws which include the increase in entropy, i. e., laws which include the irreversibility of the processes occurring in shock waves.

From this point of view shock waves afford a considerable logical and scientific interest, irrespective of their application.

It is worth noting that all basic relations and fundamental concepts have been established from the study of the general equations of gas dynamics some 50 years ago, at a time, that is, when there existed no experimental material, and long before shock waves were investigated by researchers.

As Emile Jouguet once said in a very poignant figure of speech, "the shock waves first appeared on the point of the pen of a theoretician."

We cannot but marvel at the keen analysis and theoretizing power of the great minds of the past century, first of all of the German mathematician Bernhard Riemann, the English physicist Rankine and the French artillerist Hugoniot; from different approaches and independently of one another they have created the theory of shock waves which, to this day, has not lost its significance.

Finally, the interest in shock waves has increased over recent years in connection with the problem of the destructive effect of explosions and the propagation of the explosion on the explosive substance (capable of chemical reaction). It is necessary to know exactly the condition of the substance compressed by the shock wave, the rate of compression and similar properties of the wave. The present book is an introduction to the theory of explosions.

It is the author's pleasant duty to express his gratitude to Prof. N. N. Andreyev, B. P. Konstantinov, L. D. Landau, M. A. Sadovskiy, O. M. Todes and Yu. B. Khariton for going over his manuscript and giving valuable advice.

Literature: Popular introduction to hydrodynamics [22];* some general manuals on gas dynamics [4, 23, 23, 27, 39, 106].

*Figures in brackets correspond to the numbers of the bibliography.

Chapter 1

Gas Dynamics Equations

We set up gas dynamics equations and neglect the effect of the force of gravity and also (see below) that of viscosity and thermal conductivity. For the sake of simplicity we shall write the equation for the one-dimensional case; generalization to two and three-dimensional cases will then not be difficult.

We begin with the continuity equation, i.e., the equation that expresses the law of conservation of matter.

We denote, as usual, by d/dt the substantial derivative in time, i.e., the derivative taken for the given particle along its path, and by $\partial/\partial t$ the local derivative in time which characterizes the change of the studied quantities at the given point in space, and write

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}=-\rho\frac{\partial u}{\partial x}, \qquad \text{(I-1)}$$

or

$$\frac{\partial\rho}{\partial t}=-\frac{\partial(\rho u)}{\partial x}=-\rho\frac{\partial u}{\partial x}-u\frac{\partial\rho}{\partial x}. \qquad \text{(I-2)}$$

Both formulas are, of course, completely equivalent. To derive the first formula we observe the motion of the layer of matter that comprises a constant amount of that matter. The second formula is derived by observing the change in density at the given point in space.

The equation of motion does not differ from the equation of motion for incompressible fluids:

$$\rho\frac{du}{dt}=\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}=-\frac{\partial p}{\partial x}. \qquad \text{(I-3)}$$

Finally, the third equation is substantially new; it represents a characteristic feature of gas dynamics. This is the equation of the change of state.

In the hydromechanics of incompressible fluids we added the incompressibility equation ρ = const to the first two equations. How do we find the relation between density and pressure in a compressible fluid?

Density, pressure and temperature of a fluid are connected by an equation known as the equation of state. If we know the thermal capacity, we can connect temperature with energy. To determine the connection between density and pressure, we must set up another equation — the equation of energy of a fluid in motion. In the absence of dissipative forces (viscosity and thermal conductivity) we have

$$dE = -p\,dv;\ \frac{dE}{dt} = -p\frac{dv}{dt} = -p\frac{d\left(\frac{1}{\rho}\right)}{dt} = \frac{p}{\rho^2}\frac{d\rho}{dt}, \qquad \text{(I-4)}[1]$$

where v is specific volume, a quantity inverse to density ρ.

The energy of any element of matter under investigation can only change on account of the work of compression that is being performed on it by the surrounding volumes of the fluid (gas).

Bearing in mind the fundamental thermodynamics equation

$$dE = T\,dS - p\,dv, \qquad \text{(I-5)}[2]$$

from the energy equation we readily obtain for the studied case of the absence of dissipative forces the natural conclusion

$$T\,dS = 0;\ \frac{dS}{dt} = 0. \qquad \text{(I-6)}$$

In other words, the state of matter changes according to the adiabatic curve, it changes with constant entropy.

As is known, for an ideal gas with constant thermal capacity, the adiabatic equation is

$$p = A\rho^k, \qquad \text{(I-7)}$$

where $k = c_p/c_v$, k = const. It can also be found without considering entropy, and it was found that way in 1818 by Poisson who integrated Eq. (I-4), in which for an ideal gas we substitute Clapeyron's law

$$E = c_v T = \frac{c_v}{R} RT = \frac{c_v}{R} pv;\ dE = \frac{c_v}{R} p\,dv + \frac{c_v}{R} v\,dp. \qquad \text{(I-8)}$$

Which are the conditions of applicability of the above equations[3] in which the effect of viscosity and thermal conductivity was disregarded? It is obvious, in the first place, that in order to apply these equations the Reynolds and Peclet numbers must be high.

As is known from similarity theory and hydrodynamics of an incompressible fluid, the Reynolds number characterizes the relation of inertia and viscosity. The Peclet number plays an analogous role in that it characterizes the relation of molar heat transfer of a flowing fluid and the heat flows transferred by molecular thermal conductivity.

Thus, a high Reynolds number means that one may disregard viscosity in gas dynamics equations. A high Peclet number means that thermal conductivity may be ignored; it means that along the flow line motion takes place virtually adiabatically.

From the molecular-kinetic theory it follows that in gases the ratio of thermal conduction to volume thermal capacity (known as thermal diffusivity) is approximately equal to the viscosity to density ratio (known as kinematic viscosity). For this reason in a gas flow the Reynolds number is quite close to the Peclet number, and both conditions (namely, a high Reynolds number and a high Peclet number) coincide.

Following Karman we can give a different formulation to the condition of a high Reynolds number. We use the molecular expression for the viscosity coefficient

$$\eta = \nu\varrho = \frac{1}{3}\varrho c' l, \qquad \text{(I-9)}$$

where l is the length of the free path of the molecules in the gas, c' is the velocity of molecules, a quantity equal in magnitude to the speed of sound, and ν is kinematic viscosity (cm^2/sec).

If we substitute the expression for viscosity into the Reynolds number formula, we get

$$\mathrm{Re} = \frac{U\varrho d}{\eta} = \frac{Ud}{\nu} = 3\frac{Ud}{c' l} \approx \mathrm{Ba}\cdot\frac{d}{l} \qquad \text{(I-10)}$$

where d is the characteristic size, U is the chatacteristic velocity of the motion investigated.

The relation between the speed of motion and the speed of sound is known as the Barstow criterion

$$\frac{a}{c} = \mathrm{Ba}. \qquad \text{(I-11)}$$

In the field of gas dynamics interesting us, where the speed of motion is of the same order of magnitude as the speed of sound Ba ~ 1, the Reynolds number turns out to be of the same order of magnitude as the ratio of the dimensions of system d to the length of the molecule path 1.

The condition stated above according to which $Re \gg 1$, and according to which it is possible to ignore dissipation forces (viscosity and thermal conduction), leads to the requirement that the dimensions of the system be considerably greater than the length of the free path of molecules.

We see further, however, that the fulfillment of that condition, i.e., a system of large size, does in reality not always ensure small dissipation forces and the possibility of studying adiabatic processes only. We shall see in the following that in the presence of shock waves in a flow there occur exceedingly large gradients of all the quantities studied; the magitude of these gradients does no longer depend upon the dimension of the system, and also does not drop as the dimensions of the system increase. In these cases, we will have to consider the possibility of changing entropy no matter how large the Reynolds number is.

Generally speaking, the possibility of an increase in entropy does, in principle, depend upon the dissipation forces; all the observed large-size properties of the flow, however, and, specifically, the numerical value of entropy increase in a shock wave, do not depend upon the magnitude of viscosity and thermal conductivity (they are self-modeling with respect to thermal conductivity and viscosity); the laws of the change of state in a shock wave can thus be derived without investigating the structure of its front from the equations of conservation of matter, the amount of motion and energy, applied to the states prior and after the passage of the wave.

In the case of high Reynolds numbers, we could expect a considerable effect of turbulence. In matter of fact, however, studies of the simultaneous effect of turbulence and extremely high (of the order of the speed of sound) velocities are very few. To some extent, this lack appears to be due to the complexity of such a comparatively far-out field. On the other hand, in most typical problems of gas dynamics we are faced with short pipes and nozzles, short bodies to be flowed around; in a short pipe turbulence has no time to develop, even if the Re number is high. Finally, in the hydrodynamics of small velocities, with Ba < 1, the formation of eddies and turbulence is the only resistance mechanism for $Re \gg 1$; their consideration is absolutely necessary for studying the forces affecting a body moving in a fluid. In the case of supersonic speeds there occurs what is known as wave resistance and the possibility of irreversible dissipation of energy in steady-state shock waves; a resistance different from 0 may be found also without studying turbulence.

Appendix

In order to determine the applicability of Eq. (I-1) - (I-6), let us take the general form of gas dynamic equations (see, for instance, [23, 27]).

The equation of motion has the form:

$$\varrho \frac{du_x}{dt} = \varrho X - \frac{\partial p}{\partial x} - \frac{\partial T_{xx}}{\partial x} - \frac{\partial T_{xy}}{\partial y} - \frac{\partial T_{xz}}{\partial z}, \qquad \text{(I-12)}$$

where the quantities X, Y, Z are components of volumetric force applied to a unit of mass, and the quantities T_{xx}, T_{xy}, and so forth, are components of the tensor of stresses due to the effect of viscosity. The effect of viscosity depends on the relative motion of neighboring fluid particles. From the conditions of tensor symmetry, confining ourselves to terms proportional to the first derivatives of velocity with respect to the coordinate, taking the invariant sum of normal stresses on three mutually perpendicular platforms to be equalled to the three-fold pressure, and isolating pressure from the stress tensor, as this already has been done in formula (I-12), we arrive at the following expression for the stress tensor:

$$T_{xx} = \frac{2}{3}\eta\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) - 2\eta\frac{\partial u_x}{\partial x}; \quad T_{xz} = -\eta\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right); \qquad \text{(I-13)}$$

$$T_{xy} = -\eta\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right).$$

The equations of motion with respect to the two other coordinates are found from (I-12) and (I-13) by a cyclic shifting of indices.

The coefficients in (I-13) have been chosen such that

$$T_{xx}+T_{yy}+T_{zz}=0. \qquad \text{A11}$$

In the one-dimensional case

$$u_x=u(x),\ u_y=u_z=0,\ \frac{\partial u_x}{\partial y}=\frac{\partial u_x}{\partial z}=0 \qquad \text{B11}$$

and the equation of motion (I-12) can be simplified to

$$\varrho\frac{du_x}{dt}=\varrho X-\frac{\partial p}{\partial x}+\frac{4}{3}\frac{\partial}{\partial x}\,\eta\left(\frac{\partial u_x}{\partial x}\right). \qquad \text{(I-14)}$$

If viscosity and thermal conduction are taken into consideration, additional terms appear also in the equation of energy: in the general case of three-dimensional motion (λ is thermal conduction)

$$\varrho\frac{d\left(E+\frac{u_x^2+u_y^2+u_z^2}{2}\right)}{dt}=\varrho\,(u_xX+u_yY+u_zZ)-$$
$$-\frac{\partial}{\partial x}[u_x(p+T_{xx})+u_yT_{xy}+u_zT_{xz}]-\frac{\partial}{\partial y}[u_y(\ldots.)+\ldots]-$$
$$-\frac{\partial}{\partial z}[u_z(\ldots.)+\ldots]+\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}+\frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y}+\frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z}. \qquad \text{(I-15)}$$

We remind the reader that T without indices is absolute temperature. By using the continuity equation, the equations of motion in the form (I-12) and the thermodynamic relation $dE = -pdv + TdS$, we can transform (I-15) to the following form:

$$\varrho T\frac{dS}{dt}=-T_{xx}\frac{\partial u_x}{\partial x}-T_{xy}\frac{\partial u_y}{\partial x}-T_{xz}\frac{\partial u_z}{\partial x}-T_{yx}\frac{\partial u_x}{\partial y}-T_{yy}\frac{\partial u_y}{\partial y}-$$
$$-T_{yz}\frac{\partial u_z}{\partial y}-T_{zx}\frac{\partial u_x}{\partial z}-T_{zy}\frac{\partial u_y}{\partial z}-T_{zz}\frac{\partial u_z}{\partial z}+\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}+ \qquad \text{(I-16)}$$
$$+\frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y}+\frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z}. \qquad \text{(I-16)}$$

By substituting the expressions (I-13) of the components of the tensor of viscous stresses, we reduce the expression for the work performed by viscosity, irreversibly transforming itself into heat in (I-16), to a form which shows that this quantity is essentially positive:

$$\varrho T\frac{dS}{dt}=\eta\left\{\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)^2+\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)^2+\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right)^2+\right.$$
$$\left.+\frac{4}{3}\eta\left[\left(\frac{\partial u_x}{\partial x}-\frac{\partial u_y}{\partial y}\right)^2+\left(\frac{\partial u_y}{\partial y}-\frac{\partial u_z}{\partial z}\right)^2+\left(\frac{\partial u_z}{\partial z}-\frac{\partial u_x}{\partial x}\right)^2\right]\right\}+$$
$$+\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}+\frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y}+\frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z}. \qquad \text{(I-17)}$$

In the case of one-dimensional motion

$$\varrho T \frac{dS}{dt} = \frac{4}{3} \eta \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x}. \tag{I-18}$$

We introduce the dimensionless variables: coordinates referred to the characteristic dimension of system d, velocity referred to the characteristic velocity (mean velocity or velocity in terms of a random but definite point of the system) U, and time referred to the quantity d/U. We denote the dimensionless variables with a prime:

$$x' = x/d; \quad u' = u/U; \quad t' = tU/d. \tag{I-19}$$

We refer entropy to thermal conductivity of the gas: $S' = S/c_p$. If we switch to the dimensionless variables, we find:

$$\frac{du_x'}{dt'} = \frac{Xd}{U^2} - \frac{1}{\varrho U^2} \frac{\partial p}{\partial x'} - \frac{\eta}{\varrho U d} \left\{ \frac{4}{3} \left[\frac{\partial^2 u'}{\partial x'^2} + \frac{d\eta}{\eta\, dx'} \left(\frac{du'}{\partial x}\right)^2 \right] + \cdots \right\},$$

$$\frac{dS'}{dt'} = \frac{\varrho U^2}{c_p T} \frac{\eta}{\varrho U d} \left\{ \left[\left(\frac{du_z'}{\partial y'} + \cdots \right\} + \right.\right. \quad \text{(I-}$$

$$+ \frac{\lambda}{\varrho U c_p d} \cdot \left\{ \frac{1}{T} \frac{\partial^2 T}{\partial x'^2} + \frac{1}{T} \frac{\partial T}{\partial x'} \frac{1}{\eta} \frac{\partial \lambda}{\partial x'} + \cdots \right\}. \tag{I-20}$$

The external forces are comprised in the dimensionless equations as terms multiplied by a characteristic dimension. They can be disregarded if the motion occurs at a high speed in terms of time but is not exceedingly long in terms of space; the study of the motions of a compressible fluid in the field of gravity is the subject of dynamic meteorology and will not be touched upon in this book. The terms which describe the effect of viscosity and thermal conductivity according to the statement on page 9 (7)-(8) have the coefficients

$$\frac{\eta}{\varrho U d} = \frac{1}{\mathrm{Re}} \text{ and } \frac{\lambda}{\varrho U c_p d} = \frac{1}{\mathrm{Pe}}, \tag{I-21}$$

where Re and Pe are the Reynolds number and the Peclet number.

The assumption according to which the invariant sum of normal stresses on three mutually perpendicular platforms is not different from threefold pressure contains certain arbitrary elements. Of course, we can always determine pressure p precisely in that fashion, namely, as one-third of the sum of three normal stresses, but in actual fact we are taking a further step and make a physical assumption according to which pressure so determined for a given state of matter (definable by its composition, density, energy, entropy and temperature) does not differ in magnitude from pressure

p_{ct} measured under static conditions in a motionless gas. However, with the requirement of invariantness of the physical laws with respect to the transformation of coordinates we can readily associate the more general assumption according to which the invariant sum of stresses depends on the invariant consisting of derivatives from velocity components with respect to the coordinates. Such an invariant is the expression for velocity divergence

$$\operatorname{div} u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}.$$

Assuming that we can confine ourselves to the highest term (as this has already been done when setting up the expression for viscous stresses) we get

$$p = p_{ct}(\varrho, E) - \xi \operatorname{div} u. \qquad \text{(I-22)}$$

For a complete characteristic of the behavior of matter it is therefore necessary to assign two independent viscosity coefficients η and ξ.

In its most general form compatible with the invariantness of the equations, the expression for the tensor stresses is

$$T_{xx} = \eta'\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) - 2\eta\frac{\partial u_x}{\partial x}; \quad T_{xy} = -\eta\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right). \qquad \text{(I-23)}$$

where η' is the magnitude of dimensionality of viscosity which, as η, must be determined experimentally.

Assuming arbitrarily that $\eta' = \eta . 2/3$, we got (I-13). In the general case, without making this assumption, we obtain from (I-23) and (I-22)

$$-3\xi = 3\eta' - 2\eta. \qquad \text{(I-23a)}$$

The molecular kinetic theory readily describes and computes the first viscosity coefficient (η), which is equally essential in the presence or in the absence of compressibility. The quantity ξ is introduced on account of a "cut-off" stress in the flow, in which $u_y = u_z = 0$, $u_x = a + by$. This stress is due to an exchange in motion between the layers which slide one on top of the other with a different velocity on account of the chaotic transverse motion of molecules from one layer into the other. On the basis of these considerations, considering the layers which are at a distance equalling the length of the free path l, so that the average velocity (the velocity of mass motion u_x) differs by the quantity $(\partial u_x/\partial y)\, l$, calculating the number of molecules passing during a time unit from one layer to the other, and the amount of motion carried with them, we readily find [see (I-9)].

$$T_{xy}=\eta\frac{\partial u_x}{\partial y}\sim nc'm\frac{\partial u_x}{\partial y}l\sim\varrho c'l\frac{\partial u_x}{\partial y};\quad \eta\sim\varrho c'l, \tag{I-24}$$

where n is the number of molecules in a volume unit, m is the mass of an individual molecule, c' is the rate of molecule motion.

Which is the significance of the second viscosity coefficient ξ? ξ is a factor for the quantity div u, which by the continuity equation is identically connected with the rate of density change of a substance:

$$-\frac{1}{\varrho}\frac{d\varrho}{dt}=\operatorname{div} u. \tag{I-25}$$

Thus, ξ describes the dependence of pressure upon the rate of change in density, i. e., it describes the fact that when the volume changes the static value of pressure is not determined immediately. The case where the second viscosity coefficient ξ is of the same order of magnitude as η needs no particular explanation: such a case corresponds to the determination of static pressure of the same order of magnitude as the time of free path of molecules between two collisions, ηc.

There are some cases, however, in which abnormally high values of ξ are encountered.

In Chapter 2 we shall investigate in detail the extremely important example of the molecular mechanism of a similar behavior of matter: in the presence of internal degrees of freedom which yield additional thermal conductivity and are excitable at a comparatively slow rate, pressure at a given density and a given energy of the gas depends upon the degree of excitation of the internal stages of freedom. In the case of compression (increased energy) pressure is somewhat greater, in the case of rapid expansion it is somewhat smaller than the static values (which corresponds to equilibrium excitation). The effect of this phenomenon with slow processes can be described by formula (I-22); the more difficult it is to excite the internal degrees of freedom, and the longer their time of relaxation, the more noticeable is the effect under study at slower rates of change of state, and the larger will be the second viscosity coefficient ξ.

However, in the case of fast processes conditions are attained according to which the use of linear formulas (I-22, 23) is already inadmissible since the time for a change in state becomes comparable to or even smaller than the time of relaxation of the internal degrees of freedom. It is necessary to introduce the energy of excitation of the internal degrees of freedom in its explicit form and find its dependence on time by solving the differential equation of the kinetics of extablished equilibrium without the simplifying assumption (admissible only in the case of a slow rate of change in parameters) according to which the deviation from equilibrium is proportional to the rate of parameter variation. These problems are investigated in Chapter II (acoustics) and Chapter XIII (shock waves in a gas with delayed excitation). Treatment of the second viscosity coefficient has been performed by Leontovich and Mandel'shtam [16, 17].

Chapter 2

Principles of Acoustics. The Speed of Sound

In the introduction as well as in the preceding Chapter we have several times referred to a characteristic value of velocity, namely, the speed of sound. As we study the propagation of small turbulences, we shall show how from the equations of gas dynamics we obtain, at the limit, the equations of acoustics, and how in the equations of gas dynamics is comprised the speed of sound.

We transform the equations of gas dynamics given above taking the rate of motion u and the change in density to be small. The rate of motion is taken to be small as compared with the speed of sound, $u/c << 1$, and the changes in density and pressure are taken to be small as compared with the mean values of density and pressure, $\frac{\Delta\varrho}{\varrho} \sim \frac{\Delta p}{p} \ll 1$. The fluctuations of temperature in the wave in the gas are of the same order.

Furthermore, in the equations of motion we ignore the terms of an order higher than the first one in the expansion of the equation of state of matter by powers of $\Delta\varrho$ or Δp (they refer to the left out ones such as $\Delta p/p$); we also disregard u^2 as compared with uc (the ratio of eliminated terms to the remaining ones is equal to u/c).

The values of the amplitude of pressure in a sound of a certain intensity, given below, show irrefutably that these omissions are fully permissible in acoustics.

Density is written as follows:

$$\varrho = \varrho_0 + \varepsilon, \qquad \text{(II-1)}$$

where ϱ_0, initial density, is taken to be a constant quantity, and the change in density ε, connected with the propagation of sound or, generally, perturbations (turbulence) in the gas, we take to be a small quantity.

The equation of conservation of matter can be rewritten in the following form:

$$\frac{\partial\varepsilon}{\partial t} + u\frac{\partial\varepsilon}{\partial x} + (\varrho_0 + \varepsilon)\frac{\partial u}{\partial x} = 0. \qquad \text{(II-2)}$$

If we disregard quantities of a higher order of smallness, i.e., the products of two small quantities, we get

$$\frac{\partial s}{\partial t} = -\varrho_0 \frac{\partial u}{\partial x}. \tag{II-3}$$

If we disregard, in the same fashion, terms of a higher order of smallness in the equation of motion, we get

$$\varrho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial \varrho}\frac{\partial \varrho}{\partial x} = -\frac{\partial p}{\partial \varrho}\frac{\partial s}{\partial x}. \tag{II-4}$$

By differentiating the equation of conservation of matter with respect to time, and the equation of motion with respect to the coordinate, we obtain a final fundamental acoustics equation:

$$\frac{\partial^2 s}{\partial t^2} = \frac{\partial p}{\partial \varrho}\frac{\partial^2 s}{\partial x^2}. \tag{II-5}$$

We write

$$\frac{\partial p}{\partial \varrho} = c^2, \tag{II-5a}$$

and see that this equation may have two groups of solutions: a first group

$$s = s(x - ct);\quad \varrho = \varrho(x - ct);\quad u = u(x - ct);\quad p = p(x - ct), \tag{II-6}$$

and a second group

$$s = s(x + ct);\quad \varrho = \varrho(x + ct);\quad u = u(x + ct);\quad p = p(x + ct), \tag{II-6a}$$

which differs from the first in that under the function sign there is x + ct, instead of x - ct, everywhere. We understand c to be everywhere the positive root of $\frac{\partial p}{\partial \varrho}$, $c = +\sqrt{\frac{\partial p}{\partial \varrho}}$.

The first group of solutions in which all the quantities depend upon the combination x - ct, represents turbulance which expands toward the right, i.e., in the direction of increasing values of the coordinate x. In fact, if at an instant t_1 there occurred a certain state (ϱ_1, p_1, u_1) at a point x_1, then at the following instant t_2 this same state will occur at that point x_2 where the variable x -ct (upon which depend all the quantities ϱ_1, p_1, u_1 of the solution under investigation) has the same value

$$x_2 - ct_2 = x_1 - ct_1, \quad \text{(II-7)}$$

$$x_2 = x_1 + c(t_2 - t_1). \quad \text{(II-8)}$$

The assigned state propagates in the direction of increasing x at a velocity c, q. e. d.

By substituting this type of solution into the fundamental equations, we can readily find for this wave from (II-3)[3a]

$$-c\varepsilon' = -\varrho_0 u', \quad \text{(II-9)}$$

where the prime denotes the differentiation of function (II-6) with respect to the variable x - ct. If we assume at high values of x, i. e., way ahead in an unperturbed (nonturbulent) gas, u = 0, ϵ = 0, and $\varrho = \varrho_0$, we find for a wave propagating to the right.

$$u = \varepsilon \frac{c}{\varrho_0} = (\varrho - \varrho_0) \frac{c}{\varrho_0}. \quad \text{(II-10)}$$

The instant pressure value is also linearly connected with density and velocity:

$$p - p_0 = \frac{\partial p}{\partial \varrho}(\varrho - \varrho_0) = \varrho_0 uc. \quad \text{(II-11)}$$

Let us point out specifically that pressure is proportional to the first degree of velocity in sound; according to Bernoulli's theorem, in a steady flow we should have a considerably smaller change in pressure:

$$p = p_0 - \frac{\varrho_0 u^2}{2}. \quad \text{(II-12)}$$

Thus we draw extremely important conclusions from formulas (II-10) and (II-11): In a wave which propagates to the right, i. e., in the direction of increasing values of the coordinate x, the mass rate of motion u is positive where the substance is compressed, and is negative where the substance is diluted or rarefied and its density is less than normal.

Likewise, for the second wave in which all the quantities depend upon the combination x + ct, that is, for a wave propagating to the left, in the direction of decreasing x, we get

$$u = -\varepsilon \frac{c}{\varrho_0} = -(\varrho - \varrho_0) \frac{c}{\varrho_0}. \quad \text{(II-13)}$$

In both cases the velocity of motion is directed towards the direction of wave propagation where the substance is compressed.

If at an initial instant there is assigned an arbitary distribution of density and an arbitrary distribution of velocity of motion in space

$$t = 0; \quad \varrho = \varrho(x); \quad \varepsilon = \varepsilon(x) = \varrho(x) - \varrho_0; \quad u = u(x), \quad \text{(II-14)}$$

then for the two waves looked for: the first $\varepsilon_1 = \varepsilon_1(x - ct)$, $u_1 = u_1(x - ct)$ and the second $\varepsilon_2 = \varepsilon_2(x + ct)$, $u_2 = u_2(x + ct)$, we obtain two equations

$$\varepsilon_1(x) + \varepsilon_2(x) = \varrho(x) - \varrho_0 = \varepsilon(x), \quad \text{(II-15)}$$

$$u_1(x) + u_2(x) = \frac{\varepsilon_1(x) c}{\varrho_0} - \frac{\varepsilon_2(x) c}{\varrho_0} = u(x). \quad \text{(II-16)}$$

The second equation, (II-16), is obtained by applying (II-10) to ϵ_1 and u_1, and (II-13) to ϵ_2 and u_2. Then we immediately obtain

$$\left.\begin{aligned}
\varepsilon_1(x - ct) &= \frac{1}{2}\varepsilon(x - ct) + \frac{\varrho_0}{2c} u(x - ct);\\
u_1(x - ct) &= \frac{c}{2\varrho_0}\varepsilon(x - ct) + \frac{1}{2} u(x - ct);\\
\varepsilon_2(x + ct) &= \frac{1}{2}\varepsilon(x + ct) - \frac{\varrho_0}{2c} u(x + ct);\\
u_2(x + ct) &= -\frac{c}{2\varrho_0}\varepsilon(x + ct) + \frac{1}{2} u(x + ct).
\end{aligned}\right| \quad \text{(II-17)}$$

It is not difficult also to study the reflection of an arbitrary perturbation from a motionless (stationary) wall. To find a solution for the propagating perturbation $\varepsilon_1(x - ct)$, $u_1(x - ct)$, we add a wave which seemingly arrives from the other side of the wall and propagates in the inverse direction, that is, a counterwave $\varepsilon_2(x + ct)$, $u_2(x + ct)$.

The form of function ε_2 is determined from the condition of impermeability of the reflecting wall u = 0 for x = x_{st}, whence

$$u_1(x_{ст}, t) + u_2(x_{ст}, t) = 0, \quad \text{(II-18)}$$

and if we apply (II-10) and (II-13) we find

$$\varepsilon_2(x_{ст}, t) = \varepsilon_1(x_{ст}, t), \quad \text{(II-19)}$$

$$\begin{aligned} \varepsilon_2(x, t) &= \varepsilon_2(x + ct) = \varepsilon_1\left(x_{ст}, \left[t - \frac{x_{ст} - x}{c}\right]\right) = \\ &= \varepsilon_1\left(x_{ст}, \left[t - \frac{x_{ст} - x}{c}\right]\right) = \varepsilon_1(x - ct) = \\ &= \varepsilon_1\left(x, t - 2\frac{x_{ст} - x}{c}\right), \end{aligned} \quad \text{(II-20)}$$

$$u_2(x, t) = -u_1\left(x, t - 2\frac{x_{ст} - x}{c}\right). \quad \text{(II-20a)}$$

As should have been expected, density and velocity in the reflected wave (index 2) at the given point at the given instant of time depend upon the values of density and velocity in the dropping wave at this same point at an earlier instant of time, the interval being equal to the time required for covering the distance from the given point to the reflecting surface and back at the speed of sound.

Figure 1 shows the transformation of the assigned instant into the initial instant of an arbitrary distribution of density and velocity into two waves which move in opposing directions, and the reflection of one of them by a stationary (motionless) wall; We select, as an instant, an initial condition in which in a certain region there is an increased pressure, but otherwise the substance is at rest everywhere.

The consecutive series of graphs $a_0 b_0$, $a_1 b_1$, $a_2 b_2, \ldots$, corresponds to the instants $t = 0$, $t = t_1, \cdots$ Graphs a represent the instant distribution of density (the abscissa axis $\varrho = \varrho_0$), and graphs b show the distribution of velocity (abscissa axis u = 0).

The theory of the propagation of spherical waves in three-dimensional space is nearly as simple as the one-dimensional theory, as given in equations (II-1)-(II-20). The coordinate x will be replaced now by r, the radius, i.e., the distance measured from the symmetry center of motion. We investigate only spherical-symmetric motions in which each quantity (velocity, density and pressure) depends only on time and on the distance r from the symmetry center and is constant on the sphere of radius r, i.e., does not depend on the radius-vector angle drawn from the symmetry center with the coordinate axes. The motion of gas particles occurs only along the radii plotted from the symmetry center. For this reason there is no need to use vectorial designations.

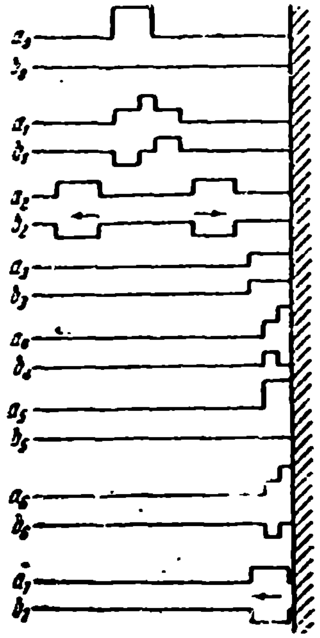

Fig. 1. Propagation and reflection of a rectilinear pressure pulse along one coordinate in linear acoustics.

The equation of conservation of matter takes the form[4]

$$\frac{\partial \varrho}{\partial t} = -\frac{\varrho_0}{r^2}\frac{\partial}{\partial r} r^2 u. \qquad \text{(II-21)}$$

The equation of motion does not change:

$$\varrho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r} = -\frac{\partial p}{\partial \varrho} \cdot \frac{\partial s}{\partial r} = \\ = -c^2 \frac{\partial s}{\partial r}. \tag{II-22}$$

By means of simple transformations we find

$$\frac{\partial^2 s}{\partial t^2} = \frac{c^2}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial s}{\partial r}. \tag{II-23}$$

In this form the equation differs from the simple equation (II-5). We substitute

$$s = \frac{\eta}{r}. \tag{II-24}$$

Then for function η, we obtain after appropriate reductions the wave equation for one-dimensional motion

$$\frac{\partial^2 \eta}{\partial t^2} = c^2 \frac{\partial^2 \eta}{\partial r^2}, \tag{II-25}$$

the solutions for which are already known

$$\eta = \eta_1 (r - ct) + \eta_2 (r + ct). \tag{II-26}$$

Thus, the general solution for the amplitude of change of density in a spherical wave takes the following form:

$$s = \frac{\eta_1 (r - ct)}{r} + \frac{\eta_2 (r + ct)}{r}. \tag{II-27}$$

By substituting expression (II-27) into Eq. (II-23), we can readily see that it satisfies the equation for arbitrary functions η_1, η_2. The first highly important difference between spherical waves and plane waves (i.e., one-dimensional waves in which all the quantities depend only on one coordinate x, (see above)) consists in that the wave amplitude during propagation from the center drops in an inversely proportional fashion to the distance from the center, see (II-27); the amplitude of a wave converging toward the center increases according to the same law. A drop in amplitude as the wave moves away from the center is perfectly natural; let us take a function η_1 such that it be different from zero only within a given interval of the change of quantity $r-ct$, $a \leqslant r-ct \leqslant b$. This means that only the substance comprised in the spherical layer of constant thickness $b-a$, $a+ct<r<a+ct+(b-a)$, is turbulent, involved in wave motion at any instant of time. As r increases with increasing time, the amount of substance involved in the motion increases proportionally to the layer volume, i.e., proportionally to r^2.

The sound energy of a volume unit is proportional to the square of the amplitude. Thus, in the absence of absorption (the transformation of sound energy into thermal energy) the law of matter conservation leads to condition $\epsilon^2 r^2 = \text{const}$, $\epsilon \sim r^{-1}$, i.e., to a decrease in the amplitude in accordance with the law mentioned above.

The second difference between spherical waves and plane waves consists in that the simple expression (II-27) is true for the amplitude of change in density and pressure, but not for velocity. Pressure and density are related by Poisson's adiabatic equation; for small amplitudes this yields

$$p-p_0=\frac{\partial p}{\partial \varrho}(\varrho-\varrho_0)=k\frac{p_0}{\varrho_0}\varepsilon=c^2\varepsilon,$$

which is exactly the same as in a plane wave. However, the simple proportionality of the speed of motion and density or pressure does not take place in the case of spherical waves (see Eq. (II-10)).

Let us substitute into (II-22) the expression of density in a spherical wave moving away from the center

$$\varepsilon=\eta_1(r-ct)\,r.$$

Then we find

$$\frac{\partial u}{\partial t}=-\frac{c^2}{\varrho_0}\left(\frac{\eta'_1(r-ct)}{r}-\frac{\eta_1(r-ct)}{r^2}\right),$$

$$u=\frac{c}{\varrho_0}\left(\frac{\eta_1(r-ct)}{r}-\frac{\int^{r-ct}\eta_1(\xi)\,d\xi}{r^2}\right)=\frac{c}{\varrho_0}\left(\varepsilon-\frac{\zeta(r-ct)}{r^2}\right). \qquad \text{(II-28)}$$

In the expression for velocity there appears an additional term which disrupts the simple proportionality of (II-10) which takes place in the propagation of plane waves. This fact leads to significant consequences which were first noted by Stokes.

Let us investigate a wave of finite width, which moves in a specific direction, namely, towards increasing coordinates; after the passage of the wave, the substance returns to its initial values of density, and then rests.

In the case of a plane wave, the dependence of density on the coordinate inside the wave (inside the region of turbulence) is not subject to any restrictions; owing to the simple relation (II-10), at the point where density returns to its initial value, velocity likewise becomes zero.

However, in the spherical case, condition $\epsilon = 0$ is not sufficient: In order that velocity become zero after the passage of the wave, it is necessary that also the second term in (II-28) become zero

$$\frac{\zeta(r-ct)}{r^2}=0; \quad \int \eta_1(\xi)\,d\xi = \int r\varepsilon\,dr = 0. \tag{II-29}$$

The integral in (II-29) is taken with respect to the entire width of the wave, i.e., with respect to the entire region in which $\epsilon \neq 0$. In formula (II-29) we can see that in a spherical wave with a finite width the change in density is bound to occur with changing signs: the integral in (II-29) will become zero only if in one of the portions of the integration region ϵ is positive and in the other it is negative. The same applies also to a change in pressure in the wave owing to a linear relation between small changes in density and pressure.

How can we represent in an elementary fashion the impossibility for a spherical wave of finite width to have compressed matter over its entire amplitude, and the causes for it ? The additional amount of matter[5] comprised in the wave is equal to $\int \varepsilon r^2 dr$. Amplitude ϵ drops as r^{-1}; thus, the additional amount of matter in a wave, in which $\epsilon > 0$ everywhere, must increase proportionally to r as the wave propagates. It is the amount of matter that increases as the wave of higher density propagates which causes a wave of lower density to follow it .

A closer examination reveals that on the borders of the wave, i.e., where both u and ϵ are very small, the quantity ζ is even smaller so that the relation between u and ϵ within the boundaries of the borders of the wave is the same as in a plane wave. Finally, it can be shown that not only a change in density but also the speed of motion u must change its sign inside the wave: there can be no spherical wave of finite width over the entire extension of which the substance would be moving in the direction of increasing radius. Inside the wave, however, the point at which the sign changes is somewhat shifted toward the symmetry center as compared with the point at which the sign of ϵ changes (Fig. 2).

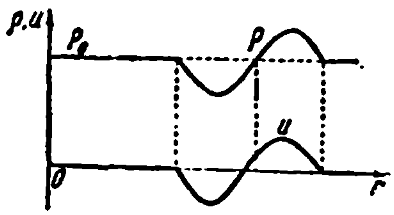

Fig. 2. Distribution of density and velocity in a spherical wave.

All this is of the greatest importance for the theory of the propagation of waves caused by an explosion, with which we shall deal in the last Chapter of this monograph.

In order to characterize the absolute values of pressure and velocities with which we have to deal in acoustics, let us give a few figures. Loudness is measured on a logarithmic scale in decibels (after the name of the inventor of the telephone, Graham Bell). An increase in loudness by n decibels (abbreviated db) means that the sound intensity increases $10^{n/10}$ times; this corresponds to an increase in the amplitude of pressure, density and velocity by $10^{n/20}$ times. Zero corresponds to the sensitivity threshold of the ear of an average person. The rustle of leaves, or whispering have a loudness of approximately 10 db, an orchestra playing fortissimo approximately 80 db (the sound intensity is 10,000,000 times greater). An extremely loud sound of 130 db produces in the air a change in density up to 0.4%, which corresponds to a pressure amplitude $p - p_0 = 0.4\% \cdot 1.4 p_0 = 0.56\%, p_0 = 56$ mm of the water column. The amplitude of the speed of motion of air particles attains 0.4% of the speed of sound, i.e., 1.3 m/sec. The amplitude of particle displacement amounts to $x - x_0 = \frac{\mu}{2\pi} \cdot 0.4\% = 0.06\%\ \mu$, i.e., 0.06% of the sound wavelength μ, about 0.036 cm for a sound with a frequency of 500 Hertz. Radiation energy equals 0.1 w/cm^2. Sound travels 330 m during m 1 sec, so that the sound energy of a volume unit at a loudness of 130 db amounts to $0.1/330 \cdot 100$ w · sec/cm^2 · cm $= 3 \cdot 10^{-6}$ J/cm^3 $= 0.7 \cdot 10^{6}$ cal/cm^3.

We point out as a comparison that the thermal energy of air under normal conditions amounts to 0.07 cal/cm^3, that is, 10^5 times greater.

Thus, not only whispering but also the fortissimo of an orchestra or the roar of a lion represent a very small shift and change in the state of the air.

The sounds perceptible to the human ear have a frequency between 20 and 20,000 Hertz (oscillations per second), i.e., a wave length from 15 m to 1.5 cm.

The speed of sound is defined by formula (II-5a).

Sir Isaac Newton in 1687 was the first to compute the absolute value of the speed of sound from the values of elasticity and density of air already known at the time, and showed the independence of the speed of sound from its amplitude and frequency. Taking the Boyle-Mariotte law for the relation between pressure and density $pv = \text{const}$, $p = \frac{\text{const}}{v} = \text{const}\ \varrho$ при $T = \text{const}$, Newton found

$$c = \sqrt{\left(\frac{\partial p}{\partial \varrho}\right)_T} = \sqrt{\frac{p}{\varrho}} = 916\,\frac{\text{feet}}{\text{sec}} = 280\,\frac{\text{m}}{\text{sec}}. \qquad \text{(II-30)}$$

Direct measurements soon showed, however, that the speed of sound in the air is almost 20% higher than the value computed by Newton. It was Laplace who explained this discrepancy in the following way: in a sound wave compression and rarefaction occur adiabatically, according to Poisson's adiabatic curve. Heating during compression and cooling during expansion enhance the changes in pressure in a sound wave, and increase its velocity

$$c^2 = \frac{\partial p}{\partial \varrho}\Big|_s = \frac{\partial (A\varrho^k)}{\partial \varrho} = k\frac{p}{\varrho}, \quad \text{(II-31)}$$

where $k = c_p/c_v$.

We bring here a table compiled by Richardson in 1939 [80] in which are juxtaposed the values of the speed of sound (in meters per second) in various media, measured experimentally and computed from the isothermic and adiabatic compressibility.

Table 1

Substance	State	T°K	c observed (m/sec)	c computed (m/sec) abiabatic	c computed (m/sec) isothermic	k
Argon	Gas	303.1	324.0	324.2	251.2	1.667
Nitrogen	Gas	273.1	337.3	336.7	284.5	1.400
Benzol	Fluid	293.1	1324	1319	1095	1.450
Teluin	Fluid	293.1	1328	1317	1138	1.340
CCl_4	Fluid	293.1	935	931	774	1.46
Water	Fluid	277	1407.0	—	—	1.000*
Water	Fluid	313	1530.3	—	—	1.026

*At 4°C, maxium density of water.

The excellent agreement with Laplace's formula proves that the change of state in a wave is strictly adiabatic. From the speed of sound Laplace found the thermal conductivity of air with constant pressure and with constant volume. Meyer ascribed the difference between c_p and c_v of the air to the work performed by the air when it expands with heating and with constant pressure. Proceeding from these considerations and from quite imprecise experimental data, Meyer approached for the first time the definition of the relationship mechanical work and heat, the "mechanical heat equivalent", the numerical basis of the law of energy conservation. Only later, under Meyer's influence, Joule performed direct experiments which confirmed the transformation of work into heat; he also found a more accurate value for the equivalent. Proceeding from the measurements of the speed of sound, Rankine computed the thermal conductivity of air in 1850, three years before the exact measurements by Ren'o (Reignaud??).

Particular mention should be made of the considerable difference between isothermic and adiabatic speeds of sound in a number of fluids. In this case the difference between c_p and c_v is no longer connected with the performance of work; instead it is connected with the increase of internal energy, with the overcoming of the cohesion of the fluid molecules with thermal expansion under constant pressure.[6]

Today the method of measuring the speed of sound is completely different from the one used at Laplace's times. His contemporaries measured with a chronograph (or a timing device) the time during which sound travels a certain distance of several kilometers. At the present time, instead, one works with short waves of strictly determined frequency mc ω is measured by an electric circuit. At a given frequency, we will find the speed of sound by determining the wavelength μ in the tested substance by the formula $c = \mu\omega$.

The wavelength is found by placing in front of the sound radiator a sound-reflecting plate which is slowly moved away from the source by means of a micrometric screw. Sound intensity reaches a maximum each time that the distance between the radiator and the reflector is travelled by an integral number of half-waves. Another maximum is reached at the same time by the consumption of energy by the radiator, recorded by electric devices.

Of great significance for physicists and chemists is the principle (thoroughly investigated in recent years) according to which the speed of sound depends on its frequency. If sound propagates in a gas in which a part of the degrees of freedom is excited at a slower rate, so that the thermal capacity of the gas depends on the rate at which the temperature changes, then we have to distinguish two critical regions. In the first region, with low vibration frequencies and a comparatively slow change in temperature, complete equilibrium has a chance to establish itself while a change in state occurs in the acoustic wave, all the degrees of freedom are excited and thermal capacity attains maximum values. In the second region, with a sufficiently rapid excitation, i.e., with a higher sound frequency, some internal degrees of freedom have no time to become excited. The change of state in the gas occurs as if its thermal capacity were smaller.

The expression for the speed of sound in a gas is

$$c^2 = k\frac{p}{\varrho}; \quad k = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}. \qquad \text{(II-31a)}$$

We see from this equation that for maximum values of thermal capacity the adiabatic index k has a minimum value, hence we obtain a minimum value for the speed of sound.

Thus, delayed excitation of the internal degrees of freedom, or of any part of thermal capacity, results in the dependence of the speed of sound on frequency, i.e., in dispersion [50].

In the case of carbon dioxide with a linear molecule (the three atoms O, C, O, are alined in equilibrium on a straight line), thermal capacity at room temperature c_v is 3.3 R. This thermal capacity is made up of progressive heat capacity 1.5 R, rotational heat capacity R and oscillatory heat capacity 0.8 R, R being the gas constant (R = 1.985 cal/degrees x mole).

Kneser's [62] measurements have shown that with frequency changing in an interval from 10^4 1/sec (10 kH) to 10^6 1/sec (1000kH), the speed of sound changes from 260 m/sec to 270 m/sec, or about 4% in accordance with the change of thermal capacity c_v from 3.3 R to 2.5 R, and the change of k from 1.3 to 1.4. It follows from these measurements that the time for establishing equilibrium in the excitation of oscillations of a CO_2 molecule is 10^{-5} sec. Oscillation is usually excited by one of 600,000 collisions, the oscillating molecule releases its energy during one of 50,000 collisions with other molecules.[7]

Analogous phenomena will take place in a system in which additional thermal capacity, excited comparatively slowly, is responsible for some reversible chemical reactions.

As an example we cite nitrogen dioxide which at room temperature is in equilibrium with nitrogen tetroxide

$$2NO_2 \rightleftarrows N_2O_4.$$

In this case, if compression time exceeds the time of the reversible reaction, we must take into account "chemical heat capacity" which arises from offset disrupted equilibrium and the release or absorption of reaction heat with changing pressure and temperature. At high frequencies, instead, equilibrium "freezes" and the system behaves as a mixture of noninert-reacting gases if the conversion of NO_2 into N_2O_4 cannot occur during an oscillation period. In 1920 Albert Einstein [50] was the first to develop the theory of sound dispersion applicable to these systems.

Simultaneously with sound dispersion, i.e., the dependence of the speed of sound on frequency, there also takes place an appreciable increase in sound absorption.

The mechanism of sound absorption can in this case be readily clarified by examining how expansion and contraction take place in the plane p, v (Fig. 3). Two adiabatic curves, BAB' and CAC', intersect at the initial point A. The first curve corresponds to rapid changes of state with a frozen part of thermal capacity, and the second one corresponds to slow equilibrium processes. If we rapidly burn the gas, it will change to state B. If we hold, with constant volume, the time required to excite the entire heat capacity, we will get to point C. In the case of rapid expansion, we will follow line CA', parallel to BA, and only after exposure for a sufficient amount of time we will again get to the initial point. Thus, the area ABCA' describes the work which, in such a cycle, has been irreversibly expended and changed into heat.[8] This work is proportional to the square of the amplitude. Here we studied a simplified cycle consisting of rapid changes of state with protracted holding in the interval. The change of state in a sinusoidal sound wave with delayed excitation of the internal degrees of freedom is described by ellipses in the plane p, v. The center of the ellipses is the point corresponding to the unperturbed state. Figure 4 shows three such ellipses. Ellipse corresponds to low frequency and slow oscillations. Motion is close to adiabatic curve CAC' (cfr. Fig. 3). The width of the ellipse, which denotes maximum deviation from equilibrium, is proportional to the rate of change of state, i.e., it is proportional to frequency ω. Consequently, also the area of the ellipse, as well as the portion of energy irreversibly converted to heat during one oscillation, are proportional

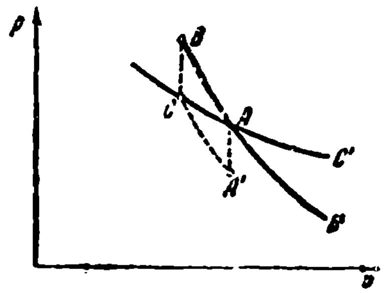

Fig. 3. Cyclic process in a gas with delayed excitation of a part of thermal capacity. Area ABCA' determines energy losses.

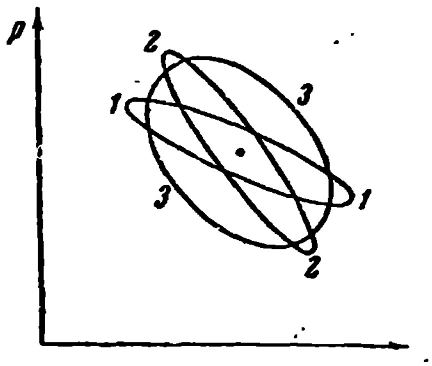

Fig. 4. Change of state with gas oscillations at delayed excitation of part of thermal capacity.

Oscillations of different frequencies: 1 - low frequency; 2 - high frequency; 3 - average frequency, oscillation period is of the same order of magnitude as thermal capacity excitation time. Ellipse area and losses per one cycle are maximal for average frequency.

to ω, hence sound absorption at a distance equal to wavelength μ is also proportional to ω. Here the behavior of matter can be described by the second viscosity coefficient (Chapter 1, Appendix). Sound absorption referred to a unit of time or a unit of length is proportional to ω^2, since oscillation time and wavelength are proportional to $1/\omega$.

In the second limiting case of extremely rapid oscillations we obtain ellipse 2; the energy of the internal degrees of freedom manages to change only by a very small value, and the entire ellipse is very close to adiabatic curve BAB'. The width of the ellipse is proportional to the amplitude of the change of energy of the internal degrees of freedom, and the amplitude, in turn, is proportional to the time during which this energy is accumulated, i.e., it is proportional to the oscillation period, and also to ω^{-1}.

The highest values of energy absorption during one oscillation are obtained with such oscillations the period of which is close to the time required for establishing equilibrium, i.e., when sound dispersion is greatest. In Fig. 4 this case is represented by ellipse 3, the width of which is of the same order of magnitude as the distance between the adiabatic curves BAB' and CAC' for maximum pressure amplitude. With slower oscillations the change of state approaches equilibrium state, and the losses during the cycle drop like ω does. With faster oscillations, the system is nearly all the time far away from the equilibrium state, excitation of internal energy occurs irreversibly, but because of the rate of the cycle it exceeds the cycle only slightly, and the losses per cycle are $\sim \omega^{-1}$.

In the second region (high frequencies), the losses referred to a unit of time tend toward a constant value. If the thermal capacity of the internal degrees of freedom is of the same order of magnitude as the entire thermal capacity, sound intensity fades to 1/e during a time equivalent to the time required to excite the internal degrees of freedom τ.

Maximum absorption and the behavior of matter at these high frequencies in the second region, where $\omega > \frac{1}{\tau}$, cannot be described by the second viscosity coefficient; they require, instead, practical concepts regarding the presence and properties of the internal degrees of freedom. A vast literature regarding dispersion and absorption of sound has become available over the recent years; in this book we can only refer to the thorough review by Richards [80].

In a system which has no delayed excitation of the internal degrees of freedom, the fundamental reasons for sound absorption are viscosity and thermal conductivity of the substance. The absorption factor on one wavelength (during one oscillation) is proportional to the frequency and inversely proportional to the wavelength μ. In the case of gases it approaches 1 as an order of magnitude, when the length of the wave approaches the length of the molecule path in the gas 1, so that we can write it as l/μ. This expression can be obtained from the exact formulas developed by Stokes [90, 91] and Kirchhoff [61] if we substitute into them the molecular-kinetic expression for viscosity (I-9) and thermal conductivity of a gas. That sounds with a wavelength smaller than the free path cannot propagate is obvious.

The effect of thermal conductivity on the propagation of sound can be explained by examining in the p, v plane the adiabatic and isothermal curves in the same way as we have examined two adiabatic curves (with and without excitation of the internal degrees of freedom). If compression occurs so rapidly that heat transfer has no chance to take place, then the change of state occurs adiabatically; in the case of slow oscillations, we can expect an isothermal change of state to take place; the transition will be accompanied by dispersion (dependence of velocity on frequency) and sound absorption.

This applies to the case of heat transfer with the outside medium, for instance, when sound propagates along a rod or in a gas enclosed in a small tube with heat-conducting walls.

If we are talking about heat transfer in a sinusoidal wave that propagates in an unlimited medium, between sections where the matter is compressed and heated and such where it is rarefied and cold, then we must bear in mind that the time of compression and expansion (the period of oscillation) is associated identically with the length of the wave.

The levelling time of the sinusoidal temperature distribution is proportional to the square of the distance, the square of the wavelength, i.e., the square of compression. Hence the apparently paradoxical conclusion according to which the signficance of heat transfer is the greater, the faster compression occurs, since by accelerating compression n times heat transfer is accelerated even more (n^2 times) and becomes considerably more substantial than in the case of slow compression.

Transition to isothermal sound propagation cannot be observed in gases, since that transition would occur at wavelengths of the order of magnitude of the free path -- where the propagation of sound is impossible; in gases, moreover, viscosity always exercises a much stronger effect than thermal conductivity.

According to Zener's most recent works [100], the levelling of thermoelastic temperature differences and the transition to isothermal propagation represent an extremely important mechanism of sound absorption in metal with a very high electron thermal conductivity. Since in a crystal the thermoelastic properties depend upon its orientation, additional losses occur in polycrystals.

It is interesting to note that in the case of the reflection by a solid wall of a sound that propagates in a gas, the temperature and velocity gradients are considerably greater than in a sinusoidal wave propagating in an unlimited space, the ratio is the greater, the smaller are viscosity and thermal conductivity, since with decreasing η and λ the depth of penetration into the gas created by the turbulence wall also decreases. In expanding these concepts, B. P. Konstantinov showed that the absorption of a sound reflected once by a wall is of the order of $\sqrt{l/\mu}$ (l being the molecule path, and μ is the length of the sound wave), i. e., it is greater by several orders of magnitude than absorption on a wavelength in the case of propagation in unlimited space [13].

Finally, let us mention the peculiar difficulties that arise in the theory of sound when examining the second approximation without neglecting compression in the wave as opposed to initial density, without neglecting mass velocity of matter motion as opposed to the velocity of sound propagation.

In this case it appears that the wave crests, i. e., the spots where density is maximal, propagate faster than the troughs, i. e., the spots where density is minimal (to the point of rarefaction). This happens for two reasons. First, in a compressed gas the speed of sound is greater because the gas temperature is higher. Second, the compressed gas has also a mass motion moving in the same direction as sound propagation; the velocity of this motion has to be added to the velocity of sound propagation. This difficulty, which is implicitly contained in Poisson's studies [75], was first noticed by Stokes in his investigations on sound propagation [92].

We can readily see from Fig. 5 that the propagating sinusoidal sound wave (a) will have to continuously change its shape.

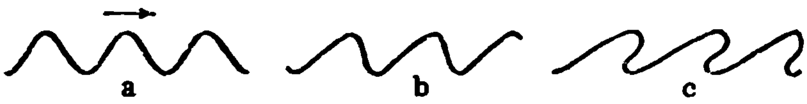

Fig. 5. Deformation of a sinusoidal sound wave as a function of propagation

a - sinusoidal wave; b - deformed wave, contains overtones; c - acoutstics equations yielded a solution devoid of physical significance, with three values for pressure of velocity at one point; in reality, however, c does not occur, shock waves are formed, dissipation forces must be calculated.

The portions of pressure increase become shorter and steeper, while the portions of pressure drop expand (b).[9] Acoustics formulas of the second approximation lead eventually to an absurd wave form (c), where at one and the same point we have three different values for density and pressure.

Analysis of this difficulty led Riemann [81] and Rankine [78] to far-reaching conclusions (see Chapter 7 and ff.)

Chapter 3

Gas Flow Through Nozzles

Let us now investigate the motion of gas in a duct with varying diameters. We will confine ourselves to a one-dimensional study of the phenomenon; we will therefore disregard the velocity components directed perpendicularly to the duct's axis, and consider all quantities (density, velocity and pressure) to be dependent only on the distance measured along the duct, but equal in any normal cross section of the duct and independent of time.

We write for the entire flow the equation of conservation of matter, which, in the case of steady flow interesting us, leads to the simple condition according to which the same amount of matter must flow during a unit of time through any cross section of the duct.

We denote by F the cross section's area and write the equation of conservation of matter in the form

$$\varrho u F = \text{const.} \qquad \text{(III-1)}$$

In the same fashion we write the equation of energy conservation which expresses the constant amount of energy flowing through a certain cross section, and the work performed there by pressure, for any cross section

$$\left(E + \frac{u^2}{2}\right) \varrho u F + p u F = \text{const.} \qquad \text{(III-2)}$$

The expression in parentheses is the energy of unit of mass, the entire first term is the energy of a unit of mass miltiplied by the amount of matter flowing during a unit of time through the entire cross section of the duct. The second term is the work performed there by pressure during a unit of time.

With the aid of the first equation, we transform the second equation to the following form:

$$I + \frac{u^2}{2} = \text{const,}^{10''} \qquad \text{(III-3)}$$

where I, known as enthalpy

$$I = E + pv, \quad \text{(III-4)}$$

is one of the fundamental functions of thermodynamics. By dividing (III-2) by (III-1) we get (III-3).

We can find the distribution of velocity and density along the pipe from the two equations above, and from the adiabatic law for the change of state of matter in a flow.

To determine the constant in Eq. (III-3), we write its value for the inlet of the pipe, i.e., for that spot where the cross section F is very large and where, accordingly, velocity u may be regarded as very low. All the quantities belonging to that cross section will be denoted by the subscript 0:

$$\left.\begin{aligned} & u_0 \rightarrow 0, \\ & I + \frac{u^2}{2} = I_0. \end{aligned}\right\} \quad \text{(III-5)}$$

We add to this the condition of adiabaticity of the flow, the absence of heat transfer to the walls and losses from hydraulic resistance. This yields for the specific entropy of matter

$$S = \text{const} = S_0. \quad \text{(III-6)}$$

Now we write the thermodynamic expression

$$dI = TdS + vdp. \quad \text{(III-7)}$$

For constant entropy

$$I - I_0 = \int_{p_0}^{p} vdp = \int_{p_0}^{p} \frac{dp}{\varrho}, \quad \text{(III-8)}$$

which together with (III-5) yields the velocity

$$\frac{u^2}{2} = -\int_{p_0}^{p} vdp = -\int_{p_0}^{p} \frac{dp}{\varrho}. \quad \text{(III-9)}$$

If the change in pressure is small, we neglect the change in the integrand

$$\frac{u^2}{2} = \frac{p_0 - p}{\varrho} = v(p_0 - p); \quad \frac{\varrho u^2}{2} = p_0 - p. \quad \text{(III-10)}$$

Equation (III-10) is then the Bernoulli law of the flow of an incompressible liquid.

If p is close to p_0, we can disregard the change in density and, as in the case of an incompressible liquid, we find that the amount of gas ϱu, that flows during a unit of time through a unit cross section is proportional to the square root of the pressure difference.

However, in the case of large pressure differentials, and with small pressure in the jet, the drop in density of the outflowing gas causes an increasing effect. Whereas the velocity increase is limited by the quantity

$$u=\sqrt{2I_0} \qquad \text{(III-11)}$$

for I = 0, gas density may drop to values as close to zero as might be desired.

Then the product ϱu becomes zero.

For a given p_0 the amount of matter flowing through a unit area of the cross section attains a maximum with a certain value of the pressure in the flow p less than p_0; it then drops again as p drops further.

We will show that maximum flow rate per unit area of cross section is attained precisely when velocity equals the speed of sound in the outflowing gas.

We seek the maximum value of the product

$$\varrho u=\varrho\sqrt{2(I_0-I)}. \qquad \text{(III-12)}$$

We take a logarithmic derivative with respect to pressure of expression (III-12) and set it equal to zero (all derivatives for S = const):

$$\frac{1}{\varrho}\frac{d\varrho}{dp}-\frac{dI/dp}{2(I_0-I)}=0, \qquad \text{(III-13)}$$

$$\frac{d\varrho}{dp}=\left(\frac{dp}{d\varrho}\right)^{-1}=c^{-2};\quad dI/dp=v=\varrho^{-1};\quad 2(I_0-I)=u^2, \qquad \text{(III-14)}$$

$$\frac{c^{-2}}{\varrho}-\frac{\varrho^{-1}}{u^2}=0;\quad c=u, \qquad \text{(III-15)}$$

q.e.d.

In an ideal gas with constant thermal capacity, the dependence of flow rate on pressure can be readily worked out analytically.

In this case the relation

$$I = c_p T = \frac{c_p}{R} RT = \frac{c_p/c_v}{(c_p - c_v)/c_v} pv = \frac{k}{k-1}\frac{p}{\varrho} = \frac{c^2}{k-1};$$

$$I_0 = \frac{c_0^2}{k-1}. \qquad \text{(III-16)}$$

holds. In an adiabatic flow

$$\varrho = \varrho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{k}}; \quad I = I_0 \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}; \quad c^2 = k\frac{p}{\varrho} = c_0^2 \cdot \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}. \qquad \text{(III-17)}$$

We introduce dimensionless variables and refer the corresponding quantities to their values at rest; the speed is referred to the speed of sound in the original gas. We denote dimensionless density by $r = \varrho/\varrho_0$, pressure by $\pi = p/p_0$, the speed of sound by $\gamma = c/c_0$, velocity by $\varphi = u/c_0$, the rate of flow per 1 cm^2 of cross section by $\psi = r\varphi = \varrho u/\varrho_0 c_0$.

Then we obtain the following equations:

$$r = \pi^{\frac{1}{k}}; \quad \gamma = \pi^{\frac{k-1}{2k}}; \quad \varphi = \sqrt{\frac{2}{k-1}\left(1 - \pi^{\frac{k-1}{k}}\right)};$$

$$\psi = \pi^{\frac{1}{k}} \sqrt{\frac{2}{k-1}\left(1 - \pi^{\frac{k-1}{k}}\right)}. \qquad \text{(III-18)}$$

Figure 6 shows the curves r, γ, φ, ψ, as functions of π for a diatomic gas (e.g., air) for which

$$k = \frac{c_p}{c_v} = \frac{7}{5} = 1.4.$$

If π changes from 1 to 0, r drops from 1 to 0, φ monotonically increases from 0 to $\sqrt{5} = 2.24$; γ drops from 1 to 0. The quantity ψ reaches the maximum of 0.58 for $\pi = 0.53$; $\psi = 0$ for $\pi = 0$ and $\pi = 1$. At the maximum point of ψ for $\pi = 0.53$, $\gamma = \varphi = 0.90$.

Using the example of air at room temperature and atmospheric pressure flowing into space with lower pressure, we will show how to use the chart in Fig. 6 plotted from dimensionless quantities. For 17°C, p_0 = 1 atm absolute, ϱ_0 of air 1.2 kg/m^3, c_0 = 340 m/sec. We find the outflow conditions for p = 0.7 atm absolute, π = 0.7. On the chart we find r = 0.785, whence ϱ = 0.785 x 1.2 = 0.93 kg/m^3; φ = 0.67, whence u = 0.67 X 340 = 227 m/sec; γ = 0.94; c = 324 m/sec. The drop in the speed of sound during outflow is the result of cooling during adiabatic expansion. Finally, ψ = 0.54, to which corresponds a flow rate per second of 0.54 x 1.2 x 340 = 220 kg/m^2 x sec.

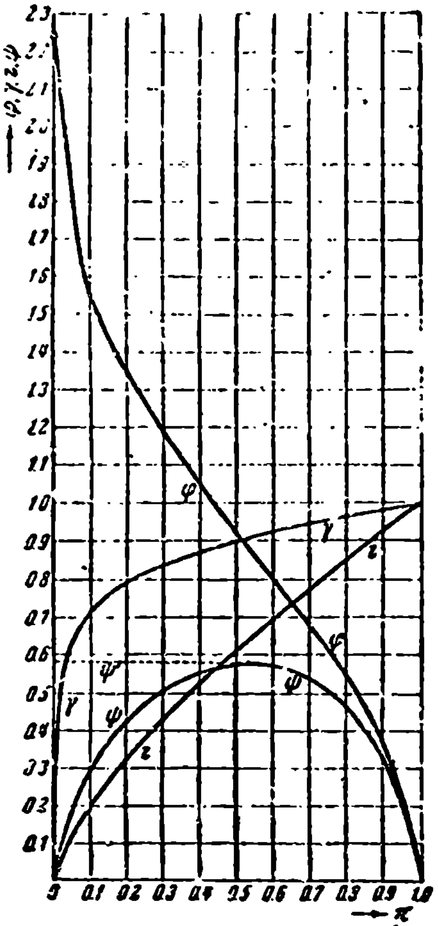

Fig. 6. Dependence of dimensionless density (r), velocity (φ), speed of sound (γ) and flow rate (ψ) on dimensionless pressure (π) in a diatomic gas with constant thermal capacity, k = 1.4 in the case of steady adiabatic outflow.

Maximum velocity of steady flow into a vacuum attains $340\sqrt{5} = 760$ m/sec.

For maximum ψ the velocity attains 360 m/sec, and the flow rate is 236 kg/m^2 x sec.

The quantities relating to the state of the gas in which maximum flow rate per unit of cross section is attained (maximum ϱu, maximum ψ) will be termed critical quantities and will be denoted by the subscript kp.

Figure 7 shows the diagram of an experimental gas flow.

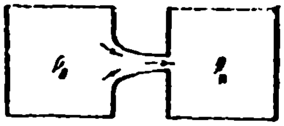

Fig. 7. Diagram of an experimental gas flow from a tapering cap (nozzle).

The vessel on the left contains a gas under pressure p_0 and is provided with a simple tapering cap (a nozzle). As the counterpressure p_n decreases in the vessel on the right into which the gas flows, the amount of outflowing gas increases according to the formula of Wentzel-St.-Venant (III-12, III-18). But if one were to follow that formula for all conditions for which the pressure in the outlet cross section of the nozzel p is taken to be equal to the pressure in the vessel on the right p_n, then, beginning with a given counterpressure, any further drop of the latter should result in a decrease of the amount of outflowing gas; specifically, for the flow into a vaccum one would reach the absurd conclusion that the rate of gas flow per second equals zero.

The fact that when the volume of outflowing substance reaches a maximum, the flow speed is exactly equal to the speed of sound (see Eq. III-15), helps explain this paradox and makes it possible to predict what will actually happen when p_n is less than p critical (i.e., p_n smaller than $0.53\ p_0$ for air).[11]

In fact, as soon as critical flow is attained, no signals can be transmitted back to the outflowing gas through the layer of gas moving at the speed of sound. If p_n is less that p_{kp}, the pressure and velocity in the nozzle will no longer change, and it will stay equal to critical pressure and critical velocity.

The amount of outflowing substance, having reached a maximum, will no longer change with smaller counterpressure values (dashed line in Fig. 6).

With a counterpressure p_n such that $p_0 > p_n > p_{kp}$, there will be an outflow regime in which the pressure p in the jet at the nozzle outlet is exactly equal to the pressure p_n in that medium into which the gas flows. The values for velocity and flow rate can be taken from Fig. 6 by substituting $\pi = p_n/p_0$.

At an appreciable distance (several nozzle diameters), the outflowing jet maintains a constant velocity along the axis, the gas particles move parallel to it at an identical speed (Fig. 8); further on the jet gradually widens and slows down as it mixes with the surrounding medium.[12]

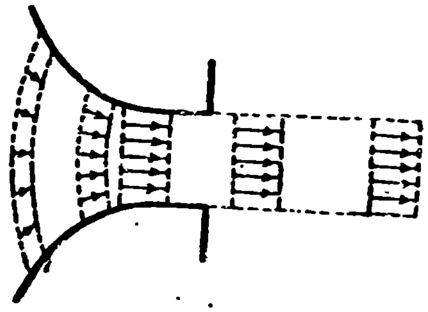

Fig. 8. Jet flow at a counterpressure exceeding critical pressure. Subcritical (subsonic) jet in free space. Pressure at jet outlet equals the pressure in surrounding medium. Speed gradually fades as jet widens due to inflow of surrounding substance.

If the counterpressure in the medium into which the gas jet flows, p_n, is smaller than critical pressure p_{kp}, the outflow conditions in the nozzle are independent of p_n.

The pressure in the nozzle's outlet cross section is equal to p_{kp} and represents a certain portion of pressure p_0 in the reservoir (slightly over one-half p_0), irrespective of the magnitude of p_n. In this case, however, the outflowing jet is not in equilibrium with the surrounding medium; the pressure difference $p_{kp} - p_n$ determines the acceleration of jet; together with velocity components directed along the axis of the nozzle, there are also radial velocity components which cause the expansion of the jet (Fig. 9). The energy of the radial velocity components cannot be exploited, hence the efficiency of the jet turns out to be less than expected with an assigned pressure differential.

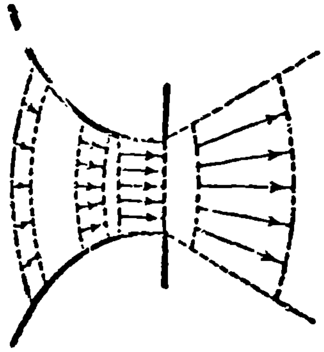

Fig. 9. Outflow of a jet from a nozzle in the presence of counterpressure less than critical. Pressure inside the jet at the outlet cross section is critical, but as the jet leaves the nozzle the pressure drops, the velocity increases and the jet expands.

The Swedish engineer Laval[13] was the first to achieve an experiment with a nozzle in which the outflow velocity of the jet exceeded the speed of sound and the jet itself had an assigned direction. In accordance with the formulas written above, when the outflow speed exceeds the critical value corresponding to the speed of sound, the flow rate per area unit ρu drops and, consequently, in order to maintain the flow rate of substance, the cross section of nozzle has to be increased (see Eq. (III-1)).

Thus Laval designed a nozzle to which his name was given, and which is shown in Fig. 10.

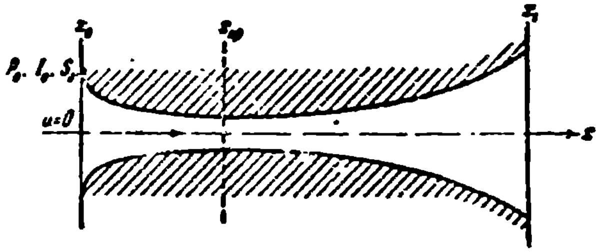

Fig. 10. Laval nozzle for obtaining directed jets at supersonic speeds

We give now another numerical example for air flow. We have a nozzle with a flow rate of 1 kg/sec at a speed of 527 m/sec. We remind ourselves of the determination of dimensionless quantities and find with the aid of Eqs. (III-18) or the diagram in Fig. 6 that for

$$\varphi = \frac{u}{c_0} = 527/340 = 1.55$$

the required counterpressure $\frac{p}{p_0} = \pi = 0.1$.

Thus, when atmospheric air flows in at p_0 = 1 atm abs, counterpressure is 0.1 atm abs. Then ψ = 0.3, and the flow rate per area unit is

$0.3\rho_0 c_0 = 0.3 \times 1.2$ kg/m^3 x 340 m/sec = 124 kg/m^2 x sec.

An assigned general flow rate of 1 kg/sec requires a cross section of the nozzle outlet of $1:124 = 0.008$ м^2 = 80 см^2, and a diameter of the circular opening of 101 mm. In the critical, narrower cross section ψ = 0.58, the flow rate is 240 kg/m^2 x sec, the cross sectional area is 42 cm^2, and the diameter is 73 mm.

We assign a specific state to the gas in the vessel whence it flows out, and then plot all the possible outflow conditions (Figs. 10 and 11) which differ by the magnitude of the gas flow rate per second A. This can be done with the aid of curve ψ from Fig. 6. For each value of the abscissa x we find in Fig. 10 the nozzle cross section F, compute the

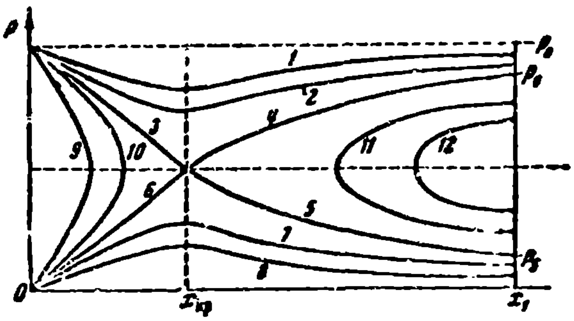

Fig. 11. Various conditions of steady adiabatic outflow in a Laval nozzle.

quantity ψ equal to $A/Fc_0\varrho_0$, and, finally, knowing ψ, we seek the corresponding values of dimensionless pressure. Since curve ψ from Fig. 6 has a maximum, then at an arbitrarily assigned value of ψ we will have, as a rule, either two values of π, or none.

If we choose a small flow rate A such that $A < F_{cr} \psi_{cr} \varrho_0 c_0$ we obtain a pair of curves, for instance, 1 and 8, or 2 and 7. The bottom curves 7 and 8 can be plotted only if a gas jet already moving at supersonic speed enters the nozzle on the left, which contradicts the assigned steady-state condition for x = 0.

The top curves 1 and 2 are perfectly reasonable solutions which can actually be obtained when counterpressure ranges in the interval $p_0 - p_4$. Qualitatively there is no difference between this motion and the one in a Venturi tube; the wider part of the Laval nozzle acts as a diffusor that restores a part of the kinetic head of the fluid. Attempts to plot conditions with a flow rate greater than critical, $A > F_{cr} \psi_{cr} \varrho_0 c_0$, lead to no solution in the middle of the tube. The corresponding pairs of curves, 10 and 11, 9 and 12, do not reflect any real motion of a fluid.

Finally, in the case of critical flow rate $A = F_{cr} \psi_{cr} \varrho_0 c_0$ the segment of curve 3 issuing from the initial point p_0, x_0 hits at the critical cross section the ramification point. With a counterpressure $p = p_4$, there will be curve 3-4 which is very close to the subsonic conditions 1 and 2.

With a counterpressure $p_n = p_5$, we have line 3-5, and the Laval nozzle yields a supersonic flow.

A further decrease in pressure cannot change the motion in the nozzle. If $p_a < p_5$ we have again line 3-5 in the nozzle and consequent expansion at the outlet.

We are unable to say, however, what happens if counterpressure ranges in the interval between p_4 and p_5. To find the answer we have first to investigate the theory of shock waves (see Chapter 18). One-dimensional theory no longer includes the design of a nozzle that would give a strictly uniform flow. For this problem, see, e.g., Busemann's paper [40].

Chapter 4

Properties of Supersonic Jets

In the preceding chapter we dealt with the theory of the Laval nozzle which makes it possible to obtain a steady parallel gas flow that moves at a supersonic spe .

Since the time Laval invented his nozzle, a considerable number of investigations were conducted into the properties of supersonic flows, which, in many respects, differ appreciably from gas flows that move at subsonic speeds.

According to a remark by Prandtl, the supersonic flow blindly runs into an obstacle. This means that the turbulence caused by an obstacle has no time to expand forward, has no time to warn the fluid particles that move toward the obstacle of what is going to happen to them; thus, the nature of the flow around obstacles, the nature of the cm supersonic flows is completely different from the customary picture of the motion of an incompressible fluid.[14]

To explain the above, let us first conduct the following simple test: beginning at a specific instant of time, we shall at specific, identical intervals produce at a given point in a flow a certain minor disturbance; in a gas at rest, this disturbance would generate spherical waves which would propagate at a speed equal to that of sound; in a gas flow, to the propagation speed of the spherical waves there will be added the speed of the flow as a whole, in other words, the spherical areas of turbulence are levelled by the flow; however, there will arise two completely different situations depending on whether the flow moves at a supersonic or subsonic speed.

In Fig. 12 (a and b) turbulence is produced at identical time intervals τ at point 0 of each diagram. In Fig. 12b, during the time τ the flow covers a distance $u\tau = 2.5$ cm; in Fig. 12a, where the flow velocity is less, the distance $u\tau = 1.5$ cm. The speed of sound c is in both cases identical and such that $c\tau = 2$ cm.[15] In a gas at rest we would have obtained a number of concentric spheres R_1, R_2, R_3; the radius of each subsequent sphere is larger than the radius of the preceding one by 2 cm. Figs. 12a and b show how these spherical surfaces are levelled by the flow.

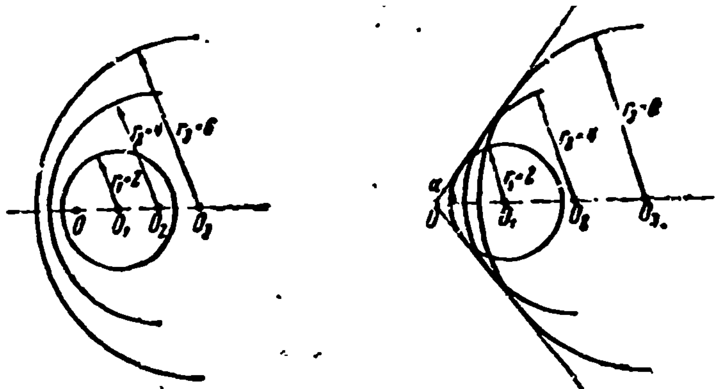

Fig. 12. Propagation of turbulence from a source in a flow moving at subsonic speed (a), and at supersonic speed (b).

In a flow moving at a subsonic speed, turbulence may move against the direction of the flow and, thus, the whole flow will gradually become turbulent: turbulence involves the entire area in which the fluid moves (Fig. 12a).

From Fig. 12b it can be seen that in a supersonic flow, turbulence envelops only a portion of the space enclosed within the cone of revolution. The angle of this cone can be readily found. As can be seen from the diagram, sin α (where α is the central angle of the cone) is equal to c/u. If the source of turbulence is an object placed within the gas flow moving at subsonic velocity, we have the usual picture of a flow around the obstacle; the velocity of the entire flow obviously differs from the flow velocity that would have existed had there been no obstacle. The turbulence caused by the obstacle expands gradually to the entire flow, and then fades out to become zero at a considerable distance from the obstacle. In a supersonic flow, the turbulence caused by the obstacle differs from zero only within the cone with the central angle found above (for the motion in the immediate vicinity of the flow around body, where turbulence cannot be considered negligible, see Chapter 17).

Thus we obtain a picture (characteristic for supersonic flows) of steady sound waves moving from any obstacle placed into or turbulence occurring within a supersonic flow.

These waves, known as Mach waves (from the name of the famous Viennese physicist who investigated them) make it possible to readily determine the velocity of a flow or, conversely, to determine the velocity of a body in a stationary gas by measuring the angle formed by the wave with the direction of motion (known as the Mach angle). Speaking generally, if the speed of sound of the gas investigated is unknown, then, in any event, the observation of the Mach waves and the measurement of the angle between then will make it possible to find at least one relationship, namely, the ratio of the velocity of the gas investigated to the speed of sound.

In these cases, however, where the state of the gas at a given point in the flow is unknown, one usually knows its "state at rest", i.e., the state in the vessel whence the gas flows out and where the gas velocity is small or negligible. The Bernoulli equation and the Mach angle equation are sufficient for determining two quantities, viz., sound velocity and flow velocity

$$i_0 = \frac{c^2}{k-1} + \frac{u^2}{2}; \quad \sin\alpha = \frac{c}{u}, \qquad \text{(IV-1)}^{16}$$

whence

$$c^2 = i_0 \frac{2(k-1)\sin^2\alpha}{k-1+2\sin^2\alpha} = c_0^2 \frac{2\sin^2\alpha}{k-1+2\sin^2\alpha}; \qquad \text{(IV-2)}$$
$$u^2 = c_0^2 \frac{2}{k-1+2\sin^2\alpha}$$

With the aid of formulas (III-18) we find the pressure and the density of the gas in the flow (assuming that entropy is constant, which is true in the absence of shock waves and in the case of a short nozzle).

There is a remarkably deep analogy between the phenomena observed in gas dynamics and the flow of a heavy, imcompressible fluid in a duct open at the top [7, 22, 73]. This analogy makes it easy to reproduce a "supersonic" fluid flow with an open surface, to perform sophisticated demonstrative tests and, in particular, demonstrate the steady propagation of waves along the surface of a fluid in the case when the fluid moves at "supersonic" speeds.

The above-mentioned analogy between a fluid with a free surface and a compressible gas is based on a simple physical phenomenon. We examine a duct open at the top into which a liquid is poured. By changing the pressure of the liquid in the duct we can change the liquid's level and thus change the amount of liquid per unit of duct bottom area, or per unit of duct length. The process of pushing upward the liquid in the duct is analogous to the process of compressing a gas contained in a pipe closed on all sides. Thus, for instance, a duct with a rectangular cross-section is equivalent to a gas governed by the Boyle-Mariette law, because in a duct with a rectangular cross-section the amount of liquid per unit of duct bottom area, i.e., what may be called density (referred to a unit of surface), is proportional to the pressure on the bottom. In the case of the motion of a liquid with an open surface, the role of sound velocity in gas dynamics is played by the propagation on the surface of a liquid of gravitational waves.

As in gas dynamics, it is possible under specific conditions to achieve a "supersonic" flow of liquid, i.e., a flow in which the speed of the liquid is greater than the propagation speed of waves over its open surface. Such a flow can be observed if we direct a water jet from a height of several tens of centimeters on a polished plane surface. Near the impact point of the jet with the surface, within a circle with a diameter of several centimeters, the layer (film) of the liquid is very thin; the liquid moves at a very high speed. If at that point we place an object, for instance, a needle, we can observe the characteristic picture of steady surface waves proceeding from the needle under a specific angle; these waves are very similar to the Mach waves in the case of a supersonic gas flow. Beyond this circle with a diameter of several centimeters, the thickness of the liquid layer abruptly increases for several millimeters; this is accompanied by a drop in the velocity of the liquid, and is in analogy with the shock wave phenomenon which x will be discussed below. In this second region, where the liquid layer is comparatively thick while the velocity of the liquid is comparatively low (less than the propagation speed of oscillations over the liquid surface), the properties of the flow are completely different.

The frequently used poetic simile of a wide river, lazily flowing along, and a mountain brook, furiously swirling over rocks and stones, is much deeper and much more significant than could be suspected. In fact, in these cases we are faced not only with a quantitative difference in the velocity of the flow. Because of the existence of a specific characteristic velocity, the velocity of wave propagation on the interface between the water and the air, the two flows (the wide river and the furious mountain brook) are also qualitiatively different.

Measuring the temperature of a supersonic flow in a Laval nozzle has yielded very interesting results. Unlike the computations from the formulas in the preceding chapter, the gas temperature measured by a thermometer or a thermocouple placed into the flow drops negligibly, and is found to be quite close to the temperature of the gas in the reservoir from which it flows out. Thus, air flowing from a reservoir in which its temperature was 300°K, must have a temperature of 250°K in the critical cross-section, and a temperature of 167°K = -106° C in the cross-section in which the flow speed is twice the speed of sound (2c). However, the temperature at this spot measured by a thermocouple is approximately 280°K. Such a result is, as a matter of fact, quite natural, because the temperature measurement with a thermometer or a thermocouple does not give the difference between thermal motion, i.e., the chaotic motion of the molecules, and mass motion of the gas, i.e., the well-organized flow. It is therefore obvious that the temperature measured by a thermometer or a thermocouple is, in reality, a gauge for the total energy of the gas, a gauge for the sum of the thermal and kinetic energy of the gas, i.e., it is the gauge for a quantity which virtually does not change in the flow. If we examine a plate placed into a flow normal to its direction, then, in examining the flow line near the plate we can see that as we approach the plate, the moving gas in experiencing a braking effect; with this, according to Bernoulli's theorem, is connected the inverse increase in pressure and, in the case of a gas, the corresponding rise in temperature to values which pressure and temperature had in the gas at rest in the reservoir from which it flows through the nozzle.[17] It is therefore obvious that the plate placed in a normal position to the flow direction, acquires not the real temperature of the moving gas, but the temperature of the slowed-down gas near the plate, which coincides with the initial temperature of the gas before it started to flow (known as its temperature at rest).

If we take a plate placed tangentially to the flow lines, then we find another reason for the increase in temperature in it; in the thin boundary layer near the plate where flow velocity changes considerably over a short distance, there occurs the release of significant amounts of heat due to internal friction in the gas. From the molecular-kinetic gas theory we can work out a ratio between the internal friction factor and the thermal conduction of the gas. The relation between effective viscosity and effective thermal conduction in a turbulent flow also satisfies that equation. Owing to this ratio it becomes possible to obtain in the general form relation between release and the removal of heat in a boundary layer.

Pehlhausen's computations [74] show, in complete agreement with the experiment, that a tangentially placed plate will also acquire a temperature in the gas which will be quite close to its temperature at rest (see also [6, 31]). Some 85% to 100% of the kinetic energy will be converted to thermal energy in the boundary layer of the gas near the plate. Accordingly, the temperature of the plate oscillates between the temperature at rest and 0.85 of that temperature plus 0.15 of the real temperature of the gas[18]

$$T_{rest} \geqslant T_{plate} \geqslant 0.85 T_{rest} + 0.15\ T_{gas}. \qquad \text{(IV-3)}$$

To measure the real temperature of a gas moving at sonic or near-sonic speeds, we must resort to a method in which the thermometer moves with the gas at the same speed. A practically convenient method is the one developed recently, which measures the temperature by inverting the spectral lines. This method, however, is applicable only at comparatively high temperatures, in any event higher than 1000°C.

The problem of the temperature acquired by a surface around which flows a gas moving at a high speed, is of great technical significance since the performance and efficiency of gas turbines are today determined by the maximum temperatures to which the blades can resist. We can see that it is inadmissible to equate the temperature of the blades to that of the gas. The temperature of the blades will always be somewhat higher because of the kinetic energy of the moving gas.

Chapter 5

Gas Flow in a Long, Cylindrical Pipe

We investigate the motion of a gas in a long, cylindrical pipe provided externally with thermal insulation. Thermal insulation was introduced so that we could take the total energy of the flow to be constant in all cross-sections. However, unlike what we did when investigating the Laval nozzle, short nozzles and attachments here we shall no longer ignore the friction of the gas against the walls, i. e., the resistance to the gas flow. The joint effect of heat release, friction near the walls and heat transfer between the walls and the gas will be that the temperature of the walls does not differ from the initial gas temperature in the reservoir from which gas flows (see the preceding chapter), and, consequently, there will be no need for thermal insulation in the particular case where the gas temperature in the reservoir is room temperature.

If we introduce hydraulic resistance to gas flow, i. e., if we introduce an irreversible process of internal friction, we can no longer take the entropy of the flow to be constant, hence our results and methods will somewhat differ from the results and methods dealt with in chapter 3.

We set up the equations for the motion under study, assuming the cross-section of the pipe to be constant. We take the complete gas flow through any cross-section of the pipe to be constant and obtain the first equation:

$$\varrho u = M = \text{const.} \qquad \text{(V-1)}$$

Also constant is the complete energy flow (plus the work of pressure forces) referred to a unit of pipe cross-section,

$$pu + \varrho u E + \frac{\varrho u^3}{2} = \text{const.} \qquad \text{(V-2)}$$

But since the amount of substance flowing through is also constant, then by dividing the second equation by the first one we obtain the constancy of the sum of enthalpy I and the kinetic energy of a unit of mass in the flow:

$$I+\frac{u^2}{2}=\text{const}=I_0 \qquad \text{(V-3)}$$

Here, as before, we denote by I_0 the enthalpy of the gas before entering the pipe, i.e., in the reservoir, where gas velocity is very low.

It is worth noting that from two equations, the equation of conservation of matter and the equation of conservation of energy, we can eliminate velocity and thus obtain a specific relation between the quantities characterizing the state of the gas, (pressure and volume); this relation is such that it does not depend upon the mechanism and the magnitude of friction [51, 89]. This can be represented graphically by curves in the plane p, v or by curves in the plane I, S, known as Fanno lines (Fig. 14).

Only the velocity of a point that represents the state of the substance moving along a Fanne line will depend on the pipe resistance, i.e., on the magnitude of dissipation forces.

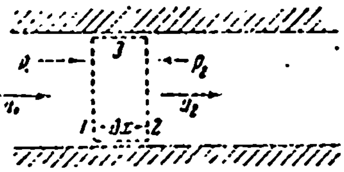

Fig. 13. An elementary cylinder cut from a long pipe. The substance flows in and out at the ends, where pressure forces are active; the lateral surface experiences the effect of friction against the pipe wall.

Let us take a portion of a long pipe Δx (Fig. 13) and clarify how over the entire stretch of Δx gas velocity and pressure change on account of resistance. The total amount of substance flowing through the pipe cross-section in a unit of time is $\varrho uF = MF = \text{const}$.

The amount of motion carried by the flow in a unit of time is $MFu = \varrho u^2 F$.

According to Newton's second law, the change in the amount of motion when covering a distance Δx between two control planes 1 and 2 is equal to the momentum of pressure

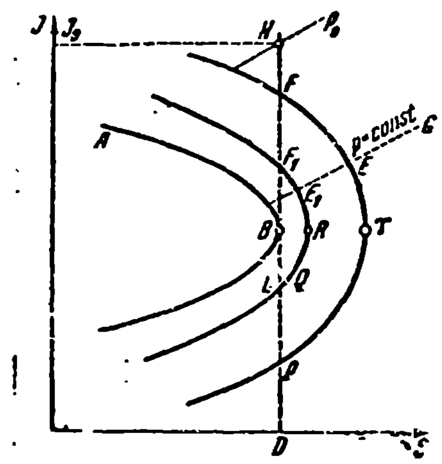

Fig. 14. Fanno lines in the entropy — enthalpy (S, I) plane. Along these lines the state of gas flowing through a pipe with a constant cross-section changes without heat transfer, but in the presence of resistance. The lines are found from the conditions of substance flow conservation and energy flow conservation in the pipe.

force (friction) Φ, acting on the lateral surface 3 of the cylinder cut by planes 1 and 2 from the pipe:

$$MF(u_2 - u_1) = (p_1 - p_2)F + \pi d \Delta x \Phi. \qquad \text{(V-4)}$$

We introduce the resistance factor in the usual way accepted by the hydrodynamics of incompressible fluids, and write for a round cylindrical pipe of diameter d the force of resistance Φ per unit of lateral surface

$$\Phi = -\zeta \rho u |u| / 8. \qquad \text{(V-5)}$$

We find from Eq. (V-4), bymaking a transition to infinitesimals and to the unit of cross-section, the equation for the amount of motion, which includes the pipes resistance. Unlike the first two equations, we cannot write it immediately in its integral form. The differential equation takes the following form

$$-\frac{d(\varrho u^2+p)}{dx}\equiv-\frac{d(Mu+p)}{dx}=\frac{\zeta}{d}\frac{\varrho u|u|}{2}. \qquad \text{(V-6)}$$

The form of the last term is somewhat different form the usual; this is due to the fact that the sign of the force of resistance depends upon the sign of velocity. The force of resistance is always directed against the direction of gas flow, and this fact is lost in the usual writing $\frac{dp}{dx}=\zeta\varrho u^2/2d$, or $\Phi=\zeta\varrho u^2/8$.

In the I, S plane, the Fanno lines corresponding to various values of flow rate M(see Eq. (V-1)), take the form shown in Fig. 14. For an ideal gas, enthalpy I coincides to an accuracy of one factor with the temperature. The I — S diagram differs from the T — S diagram only by its scale.

The quantity M is constant along each line and is a parameter that changes only from one Fanno line to the other, and decreases from left to right since for a given temperature density drops as entropy increases.

Let us now determine how a point representing the state of the gas moves along a Fanne line under the effect of resistance as the gas moves in the pipe. With the aid of the well-known thermodynamic expression for the enthalpy differential, we write the equation of conservation of energy in its differential form

$$\frac{d}{dx}\left(I+\frac{u^2}{2}\right)=\frac{dI}{dx}+\frac{u\,du}{dx}=\frac{v\,dp}{dx}+\frac{T\,dS}{dx}+\frac{u\,du}{dx}=$$
$$=\frac{1}{\varrho}\frac{dp}{dx}+\frac{T\,dS}{dx}+\frac{u\,du}{dx}=0. \qquad \text{(V-7)}$$

We substitute the value of velocity determined by the law of conservation of matter (V-1) and expressed by the quantity M constant along the entire pipe, and obtain for entropy the following equation

$$T\frac{dS}{dx}=-\frac{1}{\varrho}\left(\frac{dp}{dx}+\frac{M\,du}{dx}\right). \qquad \text{(V-8)}$$

With the aid of Eq. (V-6) we finally find

$$T\,dS=\frac{\zeta}{2d}|u|\,u\,dx. \qquad \text{(V-9)}$$

If the sign of dx coincides with the sign of flow velocity u, i.e., if, changing x, we follow the flow direction of the fluid, entropy increase is always positive since the product of udx is also positive.

The motion of substance in the presence of friction is accompanied by the conversion of mechanical energy into thermal energy; in a thermally insulated pipe, in the absence of heat take-off, this process is accompanied by an increase in the entropy of the substance flowing through the pipe.

The right-hand side of Eq. (V-9) is nothing but the work performed by the forces of resistance on an element of length dx, referred to a unit of mass of the flowing fluid.

Above the entropy maximum, on the segment AB of the Fanno line (in the subsonic region, as we shall see now), motion is accompanied by a pressure drop as in an incompressible fluid, as can be seen from juxtaposing the slope of the Fanno line and line p = const in the right-hand side of Fig. 14. Conversely, below points B, R, T, in the case of supersonic flow, resistance causes an increase in pressure along the flow; the force of resistance and the increase in pressure are overcome by the flow by means of the kinetic head, and by means of a drop in velocity due to increase in density and compression of the gas from increased pressure.

Accordingly, in a subsonic flow, u increases in the flow direction while I drops. In a supersonic flow, u drops and I grows.

Let us show that at point B of maximum entropy the flow velocity is equal to the speed of sound. This can be readily shown in Fig. 14 if we plot through B a vertical tangent. We notice that at point B, where S = maximum for M = const (motion along the Fanno line), there also takes place M = maximum for S = const (motion along the tangent). The latter condition leads to an equality between flow rate and speed of sound, as was shown in Chapter 3 (Eqs. (III-12 — III-V)).

Incidentally, the proof can easily be given directly: near point B, it is obvious that

$$\left(\frac{dS}{d\rho}\right)_{Fanno} \to 0; \quad \left(\frac{dp}{d\rho}\right)_{Fanno} \to \left(\frac{dp}{d\rho}\right)_{S=const} = c^2. \qquad \text{(V-10)}$$

From the countinuity equation (V-1) it follows that

$$\frac{du}{u} + \frac{d\rho}{\rho} = 0; \quad du = -u\frac{d\rho}{\rho}. \tag{V-11}$$

In Eq. (V-7), if we pass from differentiating with respect to coordinate x to differentiating with respect to density ρ we get at the point of tangency

$$\frac{1}{\rho}\frac{dp}{d\rho} + T\frac{dS}{d\rho} + u\frac{du}{d\rho} = \frac{1}{\rho}c^2 - \frac{1}{\rho}u^2 = 0;$$
$$u^2 = c^2, \tag{V-12}$$

q. e. d.

Now we can easily plot a physical diagram of gas flowing through long pipes. Figure 15 shows the flow rate M of gas during a unit of time as a function of pressure at the end of the pipe p, with an assigned pressure p_0 in the reservoir from which the gas is flowing out.

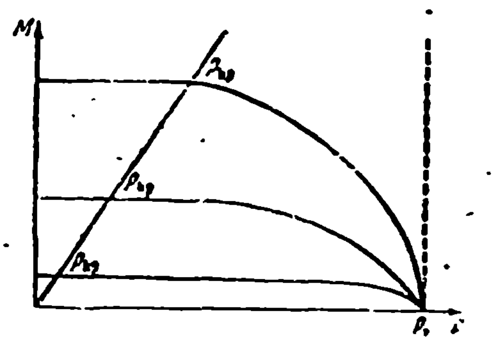

Fig. 15. Dependence of flow rate (M) on counterpressure (p) for pipes of varying length with a given pressure at the inlet (p_0). The top curve is for a short nozzle, and the bottom curve is for the longest pipe. The straight line divides on the left the region of critical outflow at a velocity equal to the speed of sound at the outlet; if counterpressure is less than critical, M does not depend on p.

The various curves are referred to pipes of different lengths. The top curve represents the case of a short nozzle as dealt with at the beginning of Chapter 3. The longer the pipe, the smaller the amount of gas flowing through it (for a given pressure difference). In all cases, a pressure drop below a certain critical value no longer causes an increase in gas flow. However, this critical pressure itself is all the smaller, the longer the pipe. For critical outflow at the outlet of the pipe, in all cases the velocity is equal to the speed of sound; the relation between the temperature of the gas and its initial temperature in the reservoir, as well as the relationship between gas velocity and sound velocity in the initial gas in the reservoir are also invariable, irrespective of the length of the pipe. However, the density of the outflowing gas, which, for a given gas temperature is proportional to the pressure, varies in accordance with the length of the pipe. Thus, the critical points for various pipes on Fig. 3 can be connected by a straight lines issuing from the origin of the coordinates. According to Stodola, for usual values of the resistance factor of commercial pipes, the critical (maximum) M when changing from short nozzles to pipes of a length of 360 diameters drops by one-half, for pipes 1000 diameters long it drops by one-third, and for pipes 5000 diameters long, it drops by one-sixth.

No matter how much we reduce pressure at the outlet from a cylindrical pipe, we will never be able to achieve supersonic speeds in the pipe. In order to accomplish this, the gas must enter the pipe already at a supersonic speed.

In the I-S diagram in Fig. 14, the inflow of the gas from the reservoir into the pipe through a short connecting nozzle AB[19] (Fig. 16a) is not described by a Fanne line but by an adiabatic curve which slopes vertically from point N (Fig. 14) and describes the initial state of the substance. In a simple tapering nozzle, the state of the substance at the inlet to the pipe is represented by any point on segment NB, for instance, F or F_1. The state of the substance at the outlet from the pipe is determined by the assigned counter-pressure p; the point representing it must be on the isobar EE_1. The selection of the Fanne line along which we change from the adiabatic curve NB to the isobar, and that corresponding to the magnitude of the gas flow for given p_0 and p, depends on the length of the pipe, and also depends on the increase of entropy along the pipe. If we increase the length of the pipe, we change from mode NF_1E_1 to mode NFE, and the flow decreases.

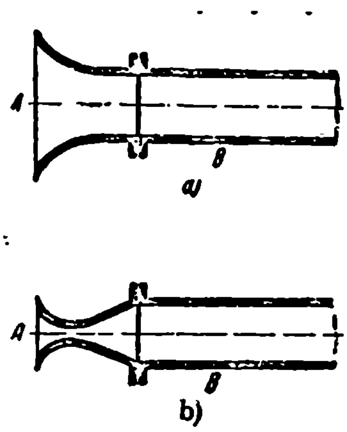

Fig. 16. Connection of the pipe with a tapering nozzle (a) and with a Laval nozzle (b). A supersonic flow inside the pipe can be obtained only in the latter case.

If counterpressure at the pipe outlet is less than critical, there will be a critical outflow, the mode described by segments NF_1E_1R or NFET (depending on the length of the pipe), with the subsequent expansion of the gas, see Chapter 3, Fig. 9.

If at the pipe inlet we place a Laval nozzle (Fig. 16b), then at the inlet we will achieve supersonic speed, we will achieve the state represented by a point on segment BD, Fig. 14, for instance, L.

In obtaining a supersonic flow, there are in the Laval nozzle outflow conditions with a fully established flow rate M (see Chapter 3); the position of point L on the segment BD can be readily determined by means of design data, viz., the cross-section of the nozzle at its narrowest point and the cross-section of the pipe.

Then, along the pipe there occurs a motion from point L to the right following the Fanno line. In the case of supersonic flow, the outflow conditions are independent of counterpressure p.

The supersonic flow conditions as shown in Fig. 16b require a sufficiently low counterpressure. However, in a long pipe it is possible that the increase of entropy along the line LQR will run into the critical point R.

Thus, in the case of a long pipe with considerable counterpressure, provided with a Laval nozzle at the inlet, we will never achieve supersonic speeds at the outlet of the pipe irrespective of the magnitude of counterpressure. A close investigation of the outflow conditions shows that in the pipe or in the nozzle there occurs what is known as a "densification jump", i. e., a shock wave, the theory of which will be discussed below. The description of the various flow conditions in the pipe in the presence of shock waves is analogous to the Laval nozzle theory (see Chapter 19). Here we can refer only to Busemann's paper [41]. A detailed bibliography, complete through 1958, can be found in Frankl, Khristianovich and Alekseyeva [27].

Chapter 6

Motion that Depends on the Relation Between Coordinates and Time

It was mentioned in the Introduction that in gas dynamics a fundamental constant of a substance in motion is a certain velocity, the velocity of propagation of turbulence, the speed of sound, etc. If we neglect the dissipation processes, matter has neither a characteristic length nor a characteristic time. From the molecular-kinetic gas theory it follows that by introducing dissipative forces, such as viscosity or thermal conduction, in combination with the characteristic values for the speed of sound, one obtains for the characteristic values of length and time the length of the free path of molecules and the time of the free path, i.e., exceedingly small (infintesimal) values for length and time. Whence it follows that if one is not interested in infinitely small processes occurring over distances and during a time of the order of magnitude of the length and the time of the free path of molecules; if, further on, we assign initial and boundary conditions for motion that contain neither a characteristic length nor a characteristic time, then one will deal with a special, extremely important class of motion. Since the equations of motion, and the initial and boundary conditions contain only the characteristic values of velocity, but not of length or time, the independent variables themselves -- the coordinate and the time -- can appear in the solution of the equations only in a combination of dimensional velocity x/t. In other words, we expect solutions that will change but still remain self-similar (self-modelling). With the increase of time counted from the instant motion begins, the character of motion as such will not change, but there will be an increase in the scale and the size of the region involved in the motion, which will be proportional to time. Accordingly, we expect that all quantities depend only on one combination of variables x/t, so that from the study of differential equations with partial derivatives for functions with two variables (coordinates and time) we can switch to ordinary differential equations in the case of motion along one coordinate.[20]

We write these equations; we denote $\xi = x/t$ and immediately set up the transformation formulae for the new viariable:

$$\frac{\partial}{\partial x}=\frac{1}{t}\frac{d}{d\xi}; \quad \frac{\partial}{\partial t}=-\frac{x}{t^2}\frac{d}{d\xi};$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}=\frac{1}{t}(u-\xi)\frac{d}{d\xi}. \qquad \text{(VI-1)}$$

As is customary in hydrodynamics, $\frac{\partial}{\partial t}$ is the notation for a local derivative with respect to time, d/dt is a substantial (that is, for a given volume moving at a speed u) derivative.

If a certain quantity f interesting us is a function of the new variable ξ, i.e.,

$$f=f(x,t)=f\left(\frac{x}{t}\right)=f(\xi), \qquad \text{(VI-2)}$$

then we obtain the following formulas

$$\frac{\partial f}{\partial x}=\frac{1}{t}\frac{df}{d\xi}; \quad \frac{\partial f}{\partial t}=-\frac{\xi}{t}\frac{df}{d\xi}; \quad \frac{df}{dt}=\frac{u-\xi}{t}\frac{df}{d\xi}. \qquad \text{(VI-3)}$$

We transform with the aid of these formulas our fundamental equations (Chapter 1), and obtain the equation of conservation of matter and the equation of motion in the following form:

$$\frac{d\varrho}{dt}=-\varrho\frac{\partial u}{dx}\rightarrow(u-\xi)\frac{d\varrho}{d\xi}=-\varrho\frac{du}{d\xi}, \qquad \text{(VI-4)}$$

$$\varrho\frac{du}{dt}=-\frac{\partial p}{dx}\rightarrow(u-\xi)\varrho\frac{du}{d\xi}=-\frac{dp}{d\xi}. \qquad \text{(VI-5)}$$

The quantities x and t can be completely eliminated from the equations, as should have been expected.

The above equations can be satisfied following the assumption that all the quantities, u, p, and ϱ are functions only of the combination $\xi = x/t$, but not of x and t individually.

Let us now show an example of initial and boundary conditions which do not contain the quantities x, t separately. We imagine an infinite plane which begins to move at an instant $t=0$ at a steady velocity w, so that the plane coordinate $x_n = wt$, $x_n/t = w$, where $w<0$ (which means that the plane moves to the left). The gas under study is to the right of the plane and expands as the plane moves (see Fig. 18).

We are looking for a solution for our equations assuming that until the instant $t=0$ the gas has been at rest and had identical constant values for density and pressure. After the piston begins to move, when $t>0$ we set the condition according to which the gas particles adhering to the piston must move at the same speed as the piston does.

Regarding the space filled with gas in which there occurs the propagation of the turbulence caused by the piston, we assume that it is unlimited toward $x>0$; the initial conditions involve no initial value for length, and the boundary conditions are formulated only on surface of the piston where they contain only an assigned piston velocity w.

At the end of this Chapter we shall investigate separately the problem regarding the extent to which the solution that depends on x/t, which we are seeking, can be used for problems involving a finite (limited) gas-filled space.

We juxtapose the equations for the conservation of matter and the conservation of motion as written above, and obtain

$$(u-\xi)^2\frac{d\rho}{d\xi}=\frac{dp}{d\xi} \qquad \text{(VI-6)}$$

whence

$$\left[(u-\xi)^2-\frac{dp}{d\rho}\right]\frac{d\rho}{d\xi}=0. \qquad \text{(VI-7)}$$

The latter equation makes it possible to construct two forms of solution: the first, a completely trivial one, ρ = const, corresponds to p, c, ρ, u = const. i.e., to the motion of the gas as a whole; the second form requires that

$$(u-\xi)^2=\pm\sqrt{\frac{dp}{d\rho}}=\pm c, \qquad \text{(VI-8)}$$

where c is the speed of sound.

We select in the latter formula the sign $u-\xi=-c$; $\xi=c+u$, which corresponds to motion on the right of the piston, i.e., to turblence propagating towards the right.

The value of ξ, and consequently, all the values of p, ρ, u, which depend on ξ alone, are constant on the lines $\xi=c+u$, $x=(c+u)t$, on the so-called characteristic equations of gas dynamics. In the problem under study all the characteristics are straight lines issuing from the origin of the coordinates x = 0, t = 0, i. e., from the point at which turbulence was started (Fig. 17).

We use the relation $u = \xi = -c$, in which c is fully determined by the state of the substance, and transform the equations of motion (VI-4) and (VI-5) to

$$c\,d\rho = \rho\,du; \quad \rho c\,du = dp. \tag{VI-9}$$

Both equations are equivalent, since $dp = c^2 d\rho$. The connection between u, ρ, p, is the same as in an acoustic (weak) wave in Chapter 2, that propagates in a positive direction.

From here we can immediately find the connection between the velocity acquired by the gas and its state

$$u = \int_{\rho_0}^{\rho} \frac{c\,d\rho}{\rho} = \int_{p_0}^{p} \frac{dp}{\rho c}. \tag{VI-10}$$

For an ideal gas with a constant thermal capacity, we write $c_p/c_v = k$ and readily compute the integrals

$$p = p_0\left(\frac{\rho}{\rho_0}\right)^k; \quad c^2 = k\frac{p}{\rho} = c_0^2\left(\frac{\rho}{\rho_0}\right)^{k-1};$$
$$u = \frac{2kp_0}{(k-1)\rho_0}\left[\left(\frac{\rho}{\rho_0}\right)^{\frac{k-1}{2}} - 1\right]. \tag{VI-11}$$

The following solution is remarkable: bearing in mind that

$$\ln c = \frac{k-1}{2}\ln\rho + \text{const}, \quad \frac{dc}{c} = \frac{k-1}{2}\frac{d\rho}{\rho}, \tag{VI-12}$$

we get

$$u = \int \frac{c\,d\rho}{\rho} = \frac{2}{k-1}\int dc = \frac{2}{k-1}(c - c_0). \tag{VI-13}$$

In order to find the distribution in space of the quantities interesting us, i.e., the structure of the wave, we must use the algebraic relation which contains the spatial coordinate ξ, $u - \xi = -c$.

In the case of a more complex relation between p and ρ, we differentiate the latter relation with respect to ξ:

$$\frac{du}{d\xi}+\frac{dc}{d\xi}=1, \qquad \text{(VI-14)}$$

and, substituting $u = u(\rho)$, $c = c(\rho)$, we get an expression for $d\rho/d\xi$ (we might, just as well, have immediately looked for an equation for another parameter, for instance, p or c).

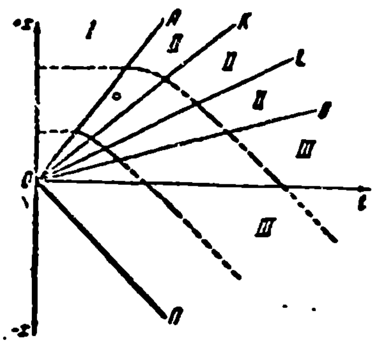

Fig. 17. Characteristics of gas-dynamic equations: the lines in the plane coordinate (x) - time (t) are OA, OK, OL, and OB. Along them are conserved all the quantities that characterize the motion and the state of a gas in the presently investigated case of turbulence caused by the movement of a piston. The pistons' movement is represented by line Π, and the motion of single gas particles is represented by the dashed lines.

In the present case of an ideal gas with a constant thermal capacity, the equations are extremely simple.

We substitute into (VI-14) $du=\frac{2}{k-1}dc$, and find

$$\frac{du}{d\xi}=\frac{2}{k+1}; \quad \frac{dc}{d\xi}=\frac{k-1}{k+1}. \qquad \text{(VI-15)}$$

The velocity of motion and the speed of sound in the wave linearly connected with the quantity ξ, which is the state propagation velocity.

With an assigned piston speed, the entire motion (Fig. 18) consists of two trivial regions: the unperturbed gas (I), and the gas that adheres to the piston and moves at a velocity which is constant in the entire region (III), and a turbulence region (II), which may be called a wave, in which all the quantities change their values in one trivial region until they reach the values in another trivial region. In each trivial (I), (III) region $\frac{dx}{d\xi}=\frac{d\rho}{d\xi}=\frac{du}{d\xi}=\frac{dc}{d\xi}=0$, $u-\xi\neq c$. Conversely, in the turbulence wave $u - \xi = - c$, and the formulas (VI-8), (VI-14), and (VI-15) apply. We can readily design a mode for any piston velocity in the case where that velocity is negative.

The distribution of velocity and pressure in space as shown in Fig. 18 corresponds to the distribution in terms of variables t, x in Fig. 17.

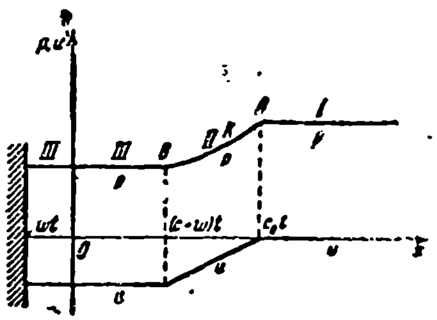

Fig. 18. Expansion wave: instant distribution of pressure p and velocity u as a function of coordinate x. As time t increases from the instant the piston begins to move, the entire distribution stretches proportionally along the abscissa. The hatched area on the left is the piston Π.

All the values along the x-axis in Fig. 18 gradually grow in accordance with the solution that depends on the ratio x/t. The wave proper is contained in the region AB(II). To the right of A we have the unperturbed gas in the state in which it was before the piston began to move (I). Between the piston and point B there is a region in which the gas moves with the same speed as the piston, the pressure and speed in the interval - B being constant ("trivial region" III). Point A moves to the right at a speed c_0. Point B moves to the right at a speed c + w, where w is the speed of the piston, that is equal to gas velocity at point B; we remind the reader that $w < 0$, and c is the speed of sound in the gas. If the piston moves at a very high speed, the quantity c - w may become negative (in the case of an ideal gas this will happen when $|w| > \frac{2}{k+1} c_0$), and point B will appear on the left of the ordinate.

At points A and B the values for velocity and pressure are continuous. Their derivatives, however, appear to be discontinuous. Hence points A and B are sometimes known as points (surface in a three-dimensional space) of weak discontinuity, or acceleration waves.

Figure 17 shows in the plane t, x the movement of the piston and the lines along which a constant value for pressure and velocity is maintained, which are known as the characteristics of the problem; these lines include such which correspond to the displacement of points A and B depending on time. Finally, the dashed lines show the trajectories of single gas particles.

In the regime under study, in which all the quantities depend on the ratio x/t alone, we proceeded from the assumption that the problem does not contain any dimensional quantities of length or time. In particular, one of the main assumptions was the unlimited stretching of the gas into the region $x > 0$.

The character of the solution found makes it possible to make this requirement less strict. If we are interested in the movement of the gas during the first t_0 seconds following the beginning of the piston's movement, turbulence (the extreme point A) will have had a change to propagate only over a distance $c_0 t_0$; for our solution to be acceptable, the second wall, the one that confines the gas on the right, be at a distance greater than $c_0 t_0$.

Thus, under any geometrical conditions, our solution is of interest for the description of the initial condition of the motion of the gas. The relation between gas velocity and pressure, and the rectilinearity of the characteristics are maintained even in the more general case involving any motion of the piston towards $x<0$ (to the left, if the gas is to the right of the piston, see Chapter 14) at a nonuniform velocity; in the case of that movement, acceleration has the same direction, $d^2x_n/dt^2<0$. This can be shown by a method of characteristics which cannot be discussed here. Equation (VI-11) is true until as a result of reflection from another wall or another turbulence the waves do not begin to propagate in the opposite direction, for which (see Formulas above, or Chapter 2) there appears another sign in expression $\rho c du = \pm dp$.

The value found by us for maximum gas velocity during its expansion is quite interesting. For an ideal gas, from our formula $-u=\frac{2}{k-1}(c_0-c)$, we see that the speed cannot exceed $-u_{max}=\frac{2}{k-1}c_0$; pressure on the piston at a speed less than critical is given by

$$\frac{p}{p_0}=\left(1-\frac{k-1}{2}\cdot\frac{-u}{c_0}\right)^{\frac{2k}{k-1}}.$$

For a diatomic gas ($c_p/c_v=1.4$) maximum velocity is equal to five times the speed of sound in the initially unperturbed gas. We can readily see that at such a speed of the piston, pressure on it is precisely equal to zero; in other words, this describes the outflow of a gas into a vacuum formerly sealed off by a partition that has been removed at a given instant (Fig. 19). For air we find $p=p_0\left(1-0.2\frac{-u}{c_0}\right)^7$.

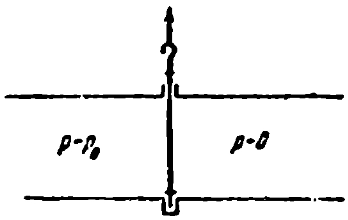

Fig. 19. Test diagram for a turbulent inflow of gas into a vacuum.

It is interesting to compare the trend of the velocity and state curves in a steady flow (Chapter 3) with those in a rarefaction wave that expands with time. In both cases the expansion of each volumetric element occurs with constant entropy, so that the relation between various quantities that characterize the state of the gas is identical:

$$S = \text{const};\quad \frac{c}{c_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{2}} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{2k}} = \left(\frac{\rho}{\rho_0}\right)^{\frac{k-1}{2}},$$

$$\text{for } k = 7/5\quad \frac{c}{c_0} = \left(\frac{T}{T_0}\right)^{1/2} = \left(\frac{p}{p_0}\right)^{1/7} = \left(\frac{\rho}{\rho_0}\right)^{1/5}.$$

As a variable that characterizes the state of the matter we conveniently choose the quantity $\gamma = c/c_0$. Velocity $\varphi = u/c_0$ referred to the initial speed of sound is expressed in the rarefaction wave (see VI-13) by the equation

$$\varphi = \frac{2}{k-1}(1-\gamma);\quad k = 1.4,\quad \varphi = 5(1-\gamma). \tag{VI-16}$$

in a steady flow (see Eqs. (III-12 and III-18))

$$\varphi = \sqrt{\frac{2}{k-1}(1-\gamma^2)};\quad k = 1.4, \varphi = \sqrt{5(1-\gamma^2)}. \tag{VI-17}$$

In Fig. 20, the last two equations for K = 1.4 are shown in solid lines. In the case of small changes of the speed of sound (for γ close to 1), i.e., in the case of slight changes of pressure, velocity in a steady flow is considerably higher than in an expansion wave. This ratio is inverted if γ is small and if pressure is small. The highest speed is obtained if steady and turbulent flow are combined, as shown by the dashed line in Fig. 20. At the point of tangency A the critical conditions for steady flow are attained, and $\varphi = \gamma$. If instead of the experiment shown in Fig. 19, we take out the plug that seals the end of the evacuated tube (Fig. 21), then at the inlet cross-section DD' there will very soon be a stationary flow (segment MA in Fig. 20), and the expansion wave (dashed line in Fig. 20) expands along the tube. Thus, under the conditions as shown in Fig. 21, it is possible to attain an even higher speed of flow into the vacuum than in the experiment shown in Fig 19. In the case of a diatomic gas, we get $5.5c_0$ instead of $5c_0$.

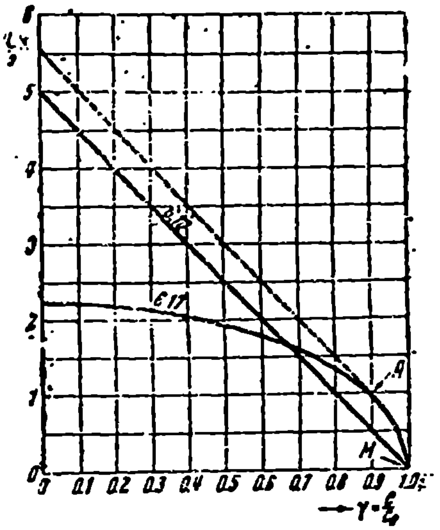

Fig. 20. Dependence of dimensionless velocity φ on dimensionless speed of sound γ for diatomic gas, k = 1.4; Eqs. (VI-17) applicable to steady flow; Eq. (VI-16) applicable to inflow as per test in Fig. 19; MA and dashed line, for inflow as per test in Fig. 21.

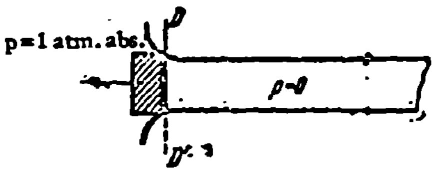

Fig. 21. Experiment of turbulent inflow of gas into a vacuum. A rounded inlet permits to obtain a higher speed than that in the experiment show in Fig. 19.

Thus, Schardin's computations [84] referred to the experiments by Craz and Schardin [44] must be corrected since he used a rounded inlet as shown in Fig. 21, whereas his computations leading to the boundary value $5c_0$ were derived from conditions as shown in Fig. 19.

Earnshaw [49] in 1860 found for the first time the numerical value for maximum flow velocity ($5c_0$). Seventeen years later it was independently found by Hugoniot in his well-known memoirs on the propagation of turbulence in a fluid [56]. In this work he points to the significance of this computation for internal ballistics. The quantity $2c_0(k - 1)$ represents obviously the maximum value of the speed of a projectile expelled by gunpowder gases in the case where the gunpowder burns up instantly and at the initial instant of the projectile's motion the gases are at rest and the speed of sound in them equals c_0 [85].

Yu. B. Khariton and this author performed detailed computations of the motion of a projectile in a gun-barrel, computed the mass of the projectile that is required to obtain an assigned speed with minimal length of the gun-barrel, taking account of the fact that gun-powder combustion products are non-ideal.

It is interesting to note that maximum flow velocity in a steady flow in considerably less -- it does not exceed

$$u'_{max}=\sqrt{2I_0}=\sqrt{\frac{2}{k-1}}c_0$$

which, in the case that $k = 1.4$, yields $u_{max}=c_0\sqrt{5}\simeq 2.2c_0$, instead of $5c_0$ in a turbulent flow. There are erroneous attempts in the literature to identify the maximum velocity of a projectile with the quantity u'_{max}, which is considerably less than the actual value (Langweiler [65]).

In the attempt to find from x/t the conditions that describe the compression of gas by a piston ($\omega > 0$), we run into a major difficulty. Our equation leads to a condition in which three values at once for velocity and pressure correspond to a number of coordinate values. In fact, as before, the equations yield $\frac{da}{d\xi} > 0$; Formally, following the same procedure as the one when studying the expansion wave, we arrive at the distribution of pressure and velocity shown in Fig. 22.

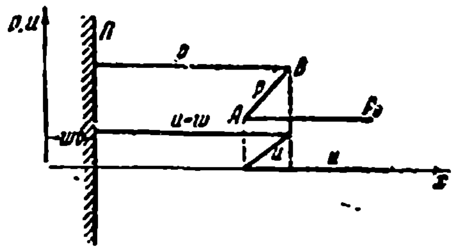

Fig. 22. Distribution of pressure and velocity having no physical significance, obtained in solving equations without dissipative forces in the case of the compression of a gas by a piston (see Fig. 18).

It is obvious that such conditions cannot be realized physically. This difficulty has inspired the theory of the shock wave which will be dealt with below.

Chapter 7

Theory of Shock Waves. Introduction

In the preceding chapters we dealt with the cases where classic gas dynamics which operates with the concept of a continuous pressure distribution and uses differential equations to describe certain phenomena, but ignores viscosity and thermal conduction, runs into certain difficulties. Let us remind the reader of the nature of these difficulties.

In the chapter on sound propagation we established that the sound wave is subject to deformation as it propagates. The "wave crests", i.e., the places where the substance is compressed and moves in the direction of wave propagation, run ahead. Conversely, the "troughs", i.e., the expansion regions where the substance moves in a direction opposite to the propagation of sound, fall behind the wave as a whole. Thus, the sound wave, as it is deformed, lashes itself -- a phenomenon similar to the one observed when sea waves run on a shallow beach.

We have mentioned several times that the analogy between gas dynamics and phenomena occurring in liquids with a free surface has a very deep and far-reaching significance. In both cases there is a tendency towards a spontaneous increase in the gradients, toward a spontaneous formation of discontinuities during compression.

In the theory of outflow from a Laval nozzle we established that it is impossible to describe a number of intermediate regimes in a specific large region of counterpressure values by means of only the equations of continuous flow with constant entropy.

Finally, in the last problem investigated by us, namely, in the case of the motion of a gas caused by the sudden movement of a piston, this limitation of classical gas dynamics became particularly obvious. Thus we have seen that if the piston moves in the same direction of the gas, $w > 0$, and the differential equations of gas dynamics lead to absurd trivalent solutions, that is, solutions according to which in one and the same spot there must simultaneously exist three values for density, three values for temperature and three values for velocity.

All these cases indicate that there must be other forms of solution in gas dynamics which are not directly derived from the equations of ideal gases (ideal here refers to the absence of viscosity and thermal conduction).

It can be expected that for the conditions sought for a large value of gradients will be characteristic, so that in a given approximation they may be treated as the propagation of the discontinuity surfaces of velocity, pressure and density -- the so-called shock waves.

Before we go into the history of the problem of shock waves, we shall derive in an elementary form the equations of a shock wave, approximately in the same way as Hugoniot in his well-known book "On the Propagation of Discontinuities" [56]. We shall postulate the existence of a discontinunity (explosion), and shall not investigate how it was achieved, whether it is steady, and so on.

Chapter 8

Hugonoit's Adiabatic Curve. Its Derivation From the Equations of Conservation

We investigate a shock wave that propagates in a gas. We are not interested here in the precise structure of the shock wave front. We only assume that even if there is no discontinuity in the strictest acceptation of that term (Fig. 23a), the changes in pressure, density, etc., do take place in a very narrow region (Fig. 23b).

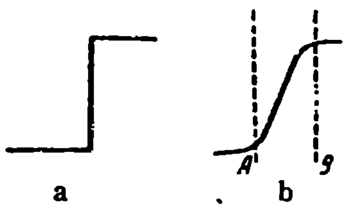

Fig. 23. The ideal (a) and actual (b) structure of a shock wave.

In our elementary derivation we shall confine ourselves to investigating the state of the substance before and after the passage of a shock wave through it. We apply the conservation equations to these states. We assume that the region proper of the wave A-B (Fig. 23b) does not increase in time. The values of pressure, density and other quantities inside the "discontinuity" itself, extended over the entire length of the segment AB, must drop out when setting up the conservation equations because although the wave travels, the amounts of matter, of energy and of motion contained in the wave between plane A and plane B are small and their change can be disregarded.

For the sake of simplicity we take a system of coordinates that travels along with the shock wave. In other words, we shall investigate a wave at rest into which through plane A there flows in matter in a state denoted by subscript 1 (on the left), and from which on the right there is an outflow of matter the parameters of which are denoted by subscript 2. We set up the conservation equations for the assigned control surfaces. We also assume that the motion of the substance occurs normal to the wave's surface.[21]

Velocity u_1, that is, the velocity at which the substance flows into the stationary shock wave, coincides obviously with the velocity at which the wave propagates with respect to the noncompressed initial substance, which is frequently denoted by D. Velocity u_2 is the wave velocity with respect to the substance compressed in the wave. Finally, the difference $u_1 - u_2$, which is independent of the choice of a moving or stationary system of coordinates, is equal to the change in gas velocity at the passage of the shock wave. In particular, in the system in which the initial substance (index 1) is at rest, the velocity after the passage of the wave

$$|u| = u_1 - u_2; \quad u_2 = D - |u|. \tag{VIII-1a}$$

If we equate the amount of substance flowing in during a unit of time to the amount of substance flowing out, we obtain the first conservation equation:

$$\rho_1 u_1 = \rho_2 u_2. \tag{VIII-1}$$

Then, for the volume enclosed between A and B we set up an expression according to Newton's second law, and equate the change in the amount of motion during a unit of time to the impulse of pressure momentum. The amount of substance $\rho_1 u_1$ flowing in during a unit of time has a velocity u_1, so that the amount of motion flowing in during a time is equal to $\rho_1 u_1^2$. The difference between the amount of motion of outflowing fluid $\rho_2 u_2^2$ and the amount of motion of inflowing fluid (i.e., the increase in momentum) must be equal to the pressure momentum which, referred to a unit of surface, amounts to $p_1 - p_2$. Thus we get the second conservation equation

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2. \tag{VIII-2}$$

Finally, we set up the equation of energy conservation. In it we will have to consider three pairs of quantities, viz., intrinsic energy of the inflowing and outflowing substance, its kinetic energy, and the work performed by pressure on the control surfaces A and B. Thus, in its definitive form, the amount of inflowing energy together with the work together with the work performed by pressure on surface A is

$$\rho_1 u_1\left(E_1+\frac{u_1^2}{2}\right)+p_1 u_1=\rho_1 u_1\left(E_1+\frac{p_1}{\rho_1}+\frac{u_1^2}{2}\right)=$$
$$=\rho_1 u_1\left(I_1+\frac{u_1^2}{2}\right). \qquad \text{(VIII-3a)}$$

This expression must be equated to an identical equation with index 2, which will give us the amount of energy carried away by the outflowing substance during a unit of time, and the work performed by the gas against the pressure on the control surface B. By cancelling the obtained equation by the quantity $\rho_1 u_1 = \rho_2 u_2$, i.e., by referring all the quantities not to a unit of shock wave surface and a unit of time, as we did before, but to a unit of mass of the substance flowing through, we obtain the third fundamental equation in the following form

$$I_1+\frac{u_1^2}{2}=I_2+\frac{u_2^2}{2}. \qquad \text{(VIII-3)}$$

Here we have again introduced enthalpy $I=E+pv=E+\frac{p}{\rho}$. All the equations are symmetrical with respect to the permutation of the subscripts 1 and 2. From the three equations we can readily eliminate the two velocities u_1 and u_2 in order to obtain the relation between pressure and density before and after the passage of the wave, which is known as the Hugoniot adiabatic equation.

From the two first equations, without using the equation of energy conservation, we find

$$\frac{u_1}{u_2}=\frac{\rho_2}{\rho_1}; \quad u_1^2=\frac{\rho_2}{\rho_1}\frac{p_2-p_1}{\rho_2-\rho_1}; \quad u_2^2=\frac{\rho_1}{\rho_2}\frac{p_1-p_2}{\rho_1-\rho_2};$$
$$u_1^2-u_2^2=\frac{(\rho_1+\rho_2)(p_1-p_2)}{\rho_1\rho_2}. \qquad \text{(VIII-4)}$$

We substitute these expressions into the last equation and obtain the Hugoniot adiabatic equation sought

$$I_1 - I_2 = \frac{1}{2\rho_1 \rho_2} (\rho_1 + \rho_2)(p_1 - p_2), \qquad \text{(VIII-5)}$$

or

$$E_1 - E_2 = \frac{1}{2\rho_1 \rho_2} (\rho_1 - \rho_2)(p_1 + p_2). \qquad \text{(VIII-6)}$$

To obtain the relation between pressure and density after compression in the wave in an explicity form, we muct express enthalpy or energy in terms of pressure and density. For an ideal gas, the thermal capacity of which we take to be constant in the temperature interval between T_1 and T_2 interesting us,

$$I = c_p T = \frac{c_p}{R} RT = \frac{c_p}{R} pv = \frac{k}{k-1} \frac{p}{\rho}, \text{[22]}$$

we obtain by means of simple transformations the relation between density and pressure for a substance passing through a discontinuity, the Hugoniot adiabatic equation

$$\frac{\rho_2}{\rho_1} = \frac{(k+1)p_2 + (k-1)p_1}{(k-1)p_2 + (k+1)p_1}; \qquad \frac{p_2}{p_1} = \frac{(k+1)\rho_2 - (k-1)\rho_1}{(k+1)\rho_1 - (k-1)\rho_2}. \qquad \text{(VIII-7)}$$

The equations can be simplified if instead of density we introduce everywhere the inverse quantity of specific volume

$$\left.\begin{aligned} &\frac{u_1}{u_2} = \frac{v_1}{v_2}; \quad u_1^2 = v_1^2 \frac{p_2 - p_1}{v_1 - v_2}; \quad u_2^2 = v_2^2 \frac{p_1 - p_2}{v_2 - v_1}; \\ &u_1 - u_2 = \sqrt{(p_1 - p_2)(v_2 - v_1)}; \\ &u_1^2 - u_2^2 = (v_1 + v_2)(p_2 - p_1); \end{aligned}\right\} \qquad \text{(VIII-8)}$$

$$\left.\begin{aligned} I_1 - I_2 &= \frac{1}{2}(v_1 + v_2)(p_1 - p_2); \\ E_1 - E_2 &= \frac{1}{2}(v_2 - v_1)(p_1 + p_2); \end{aligned}\right\} \qquad \text{(VIII-9)}$$

$$\frac{v_2}{v_1} = \frac{(k-1)p_2 + (k+1)p_1}{(k+1)p_2 + (k-1)p_1}; \qquad \frac{p_2}{p_1} = \frac{(k+1)v_1 - (k-1)v_2}{(k+1)v_2 - (k-1)v_1} \qquad \text{(VIII-10)}$$

A logically simpler derivation of Hugoniot's adiabatic equation (though physically completely equivalent to the preceding one) is the one where we proceed directly from the problem of the motion of a piston in a gas, dealt with earlier. In this case we need not operate with the concepts of energy flow and momentum flow, which may represent certain advantages for the inexperienced reader.

Let us investigate a pipe with a 1-cm^2 cross-section, closed by a piston at the origin of the coordinates. At the time instant $t = 0$ we begin to move the piston at a constant velocity w. We shall seek a motion pattern, shown in Fig. 24, where the discontinuity of all the quantities, density, velocity and pressure, propagates in front of the piston at a constant velocity D. On the right, in front of the discontinuity, the substance is completely undisturbed, and maintains its initial pressure p_1, its initial density ρ_1, and is also motionless. In the interval between the piston and the discontinuity, the substance has some other values of density ρ_2 and pressure p_2, constant over the entire interval between the piston and the discontinuity. It also moves at a velocity equal to the velocity of the piston $u = w$.

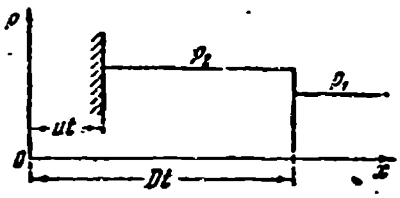

Fig. 24. Distribution of pressure in space during the passage of a shock wave caused by the compression of a gas by a piston.

We investigate the state that arises after a time t. The discontinuity moves away at a distance Dt. The amount of substance compressed during that time is $\rho_1 Dt$. It has to be equated to the amount of substance found in a gas compressed to density ρ_2 between the piston that has moved the distance ut and the discontinuity

$$\rho_1 Dt = \rho_2 (D - u) t. \qquad \text{(VIII-11)}$$

This amount of substance has acquired a velocity equal to the velocity of the piston. The total velocity acquired by the gas enclosed in the pipe during the time t is $\rho_1 Dut$. We must now equate the increase in momentum to the pressure momentum, i.e., the product of the force equal to the difference between the pressure produced by the piston and the counterpressure of the unperturbed gas during the time the force is active

$$\rho_1 Dut = (p_2 - p_1)t. \quad \text{(VIII-12)}$$

Finally, we equate the energy increase in the substance to the work performed by the piston, i.e., the work performed by the external force that moves the piston during time t. Numerically the force for a piston area of 1 cm^2 is equal to p_2, the path travelled by the piston is ut, and the work performed is p_2ut.

Thus we obtain the last equation, the energy equation

$$\rho_1 Dt\left(E_2 + \frac{u^2}{2} - E_1\right) = p_2 ut. \quad \text{(VIII-13)}$$

It is obvious that these equations are completely identical with those derived earlier, from which they can be obtained by switching to a system of coordinates having a uniform motion with respect to the system selected now. The discontinuity propagation velocity D was denoted earlier by u_1, so that now $D = u_1$, and piston velocity $u = u_1 - u_2$. The proof that the last three equations (VIII-11, VIII-12, and VIII-13) lead to the same expression for Hugoniot's adiabatic curve (VIII-5, VIII-6), is left to the reader.

Chapter 9

Properties of Hugoniot's Adiabatic Curve. Shock Waves in Air and Water.

Hugoniot's adiabatic equation derived above has a number of extremely interesting properties. First of all we can readily see that with an unbounded increase of compression pressure p_2, the density of an ideal gas with constant thermal capacity will not increase ad infinitum but will tend to a specific limit equal to $\varrho_2 = \frac{k+1}{k-1}\varrho_1$. For a diatomic gas with unexcited oscillations inside the molecule c_v = 5 cal/mole x degree; c_p = 7 cal/mole x degree; k = 1.4 and the limit value of density does not exceed initial density times 6. For a monoatomic gas the limit value for volume compression is 4.

Thus we see that in the case of strong compression, density increases rather slowly. To this corresponds a slow decrease in volume and a correspondingly rapid increase of factor pv which determines the gas temperature.

Numerical computations fully corroborate the conclusion regarding the rapid increase of gas temperature with increasing pressure in a shock wave.

Figures 25 and 26 show the curves plotted by Leypunskiy which, depending on the pressure ratio p_2/p_1, give us all the quantities interesting us -- density after compression, all the velocities u_1, u_2, $u_1 - u_2$ and sound velocity in the compressed gas. All velocities are referred to sound velocity in the initial, unperturbed gas. The temperature of the compressed gas can readily be found from the sonic velocity curve $T_2/T_1 = (c_2/c_1)^2$. The computations have been performed assuming a constant thermal capacity c_v = 5 cal/mole x degree, c_p = 7 cal/mole x degree, independent of temperature. Becker [38] gives us a table of the state of air compressed by a shock wave.

Becker conducted his computations assuming that thermal capacity is linearly dependent upon temperature. Mean thermal capacity in the interval from 273° to T is expressed by the formula

$$c_{v(273-T)} = 4.78 + 0.45 \cdot 10^{-3}\,T. \qquad \text{(IX-1)}$$

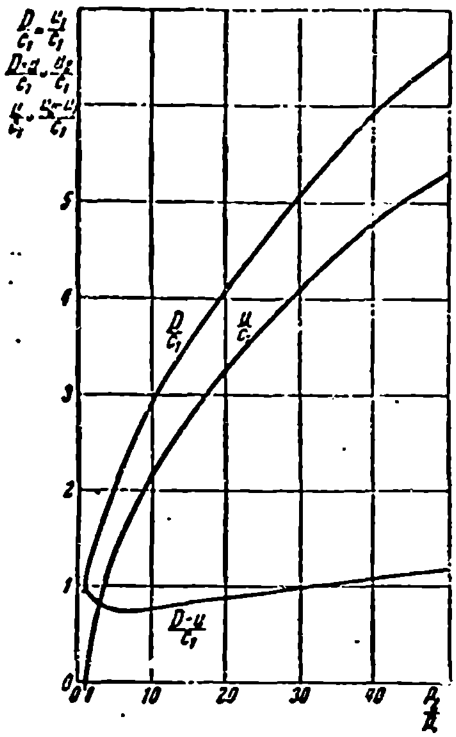

Fig. 25. Dependence of wave propagation velocity D, the velocity of compressed substance u and wave velocity with respect to compressed substance D - u, on pressure amplitude in a shock wave in a diatomic gas with constant thermal capacity.

Check computations show that in the interval from room temperature to 3000°K this simple formula coincides with an accuracy up to 3% with the modern exact value for the thermal capacity of air calculated on the basis of spectroscopic data. In his table Becker, as a comparison, gives the temperature that can be attained with adiabatic compression (along Poisson's adiabatic curve with constant entropy) up to the same pressure.

It can be seen from the Table that compression of the shock wave leads with an equal pressure increase to a considerably higher compression temperature.

Direct calculations for an ideal gas with constant thermal capacity show that with compression in the shock wave, i.e., when $\rho_2 > \rho_1$; $p_2 > p_1$; $v_2 < v_1$; $u_1 > 0$; $u_2 > 0$, the following relationships take place

$$u_1 > c_1; \quad u_2 < c_2; \quad S_2 > S_1. \tag{IX-2}$$

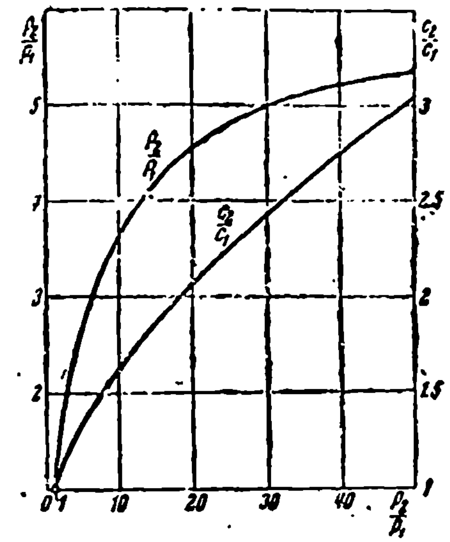

Fig. 26. Dependence of density ρ_2 and sound velocity c_2 in a compressed medium, on pressure, under the same conditions as in Fig. 25.

In an expansion wave, if it propagated in the form of a discontinuity, in an ideal gas $u_1 > 0$; $u_2 > 0$; $\rho_2 < \rho_1$; $p_2 < p_1$; the relationships would be inverted

$$u_1 < c_1; \quad u_2 > c_2; \quad S_1 < S_2. \tag{IX-3}$$

In the case of absence of heat transfer to the outside, a drop in entropy is impossible, hence it follows that a expansion wave cannot propagate in the form of a discontinuity (the so-called Zemplen theorem [99, 55 . Below, as we will go deeper into the theory of shock waves, we will show the mechanism of entropy increase in a compression wave and its connection with inequalities which refer to wave velocity u_1, u_2 and sound velocity $c_1 c_2$.

For an ideal gas with constant thermal capacity and a large shock wave amplitude $p_2 \gg p_1$, the formulas become considerably simpler. We have already noticed that density after compression has a specific relation (k + 1/k - 1) to density before compression. The relations between quantities that characterize the state achieved after compression, approach a specific limit for $\frac{p_2}{p_1} \to \infty$ (sic)

$$D : u : c_2 = k+1 : 2 : \sqrt{2k(k-1)}. \qquad \text{(IX-4)}$$

The limit formulas include initial density but not initial pressure or temperature, upon which the final state does not depend in the case of a large amplitude

$$D = \sqrt{\frac{k+1}{2} p_2 v_1} = \sqrt{\frac{k+1}{2} \frac{p_2}{\rho_1}}; \quad T = \frac{k-1}{k+1} \frac{p_2 v_1}{R} = \frac{k-1}{k+1} \frac{p_2}{R \rho_1}. \qquad \text{(IX-5)}$$

Table 2

p_2/p_1	ρ_2/ρ_1	T_2 °K	T (p, S = const)	D m/sec a)	u m/sec a)
1	1	273	273	330	0
2	1.63	336	330	452	175
5	2.84	482	426	698	452
10	3.88	705	515	978	725
50	6.04	2 260	794	2 150	1 795
100	7.66	3 860	950	3 020	2 590
500	11.15	12 200	1 433	6 570	5 980
1 000	14.3	19 100	1 710	9 210	8 560
2 000	18.8	29 000	2 070	12 900	12 210
3 000	22.3	36 700	2 180	15 750	15 050

Figures below the line are unreliable.

CODE: a) Sec

The propagation of shock waves in fluids has been scantily investigated. In his monograph on shock waves, Becker cites data on shock waves in alcohol and ether. He performed his computations with the aid of Tamman's approximate equation of state.

Considering the importance of the study of shock waves in water with regard to underwater explosions of mines and torpedoes, the evaluation of the fundamental parameters of a shock wave as a function of pressure appears to be interesting.

The next Table gives the results of the computation of the propagation of a shock wave in water performed by Leypunskiy and this author [125].

Unlike Becker, we used in our calculations the tabular data for compressibility, the expansion factor and the thermal capacity of the water, without resorting to the unreliable equations of state.

Bridgeman's measurements reached extremely high pressures, hence in the computations there was no need to use extrapolation. For the sake of convenience, the initial conditions were so chosen that the final temperature of the water compressed in the wave be 40°C. At this temperature, according to Bridgeman, the coefficient of thermal expansion of water α does not depend on pressure. This, of course, facilitates computations.

The energy equation (VIII-6) comprises the energy of water at very high pressures. We computed with the aid of the thermodynamic relations

$$dE = TdS - pdv = T\left(\frac{\partial S}{\partial T}\right)_p dT + T\left(\frac{\partial S}{\partial p}\right)_T dp - pdv,$$

$$\left(\frac{\partial S}{\partial T}\right)_p = \frac{c_p}{T}; \quad \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p = -\alpha v,$$

$$dE = c_p\, dT - \alpha Tv\, dp - pdv.$$

By integrating the last equation with respect to the path leading from a state with a certain energy to a state with energy to be determined, we find E.

Table 3

p_1	T_1, °C	v_1	c_1	p_2	T_2, °C	v_2	c_2	u	D	$T'_{1\sigma}$, °C
1	40	1.008	1530	1	40	1.008	1530	0	1530	40
1	31.5	1.005	1500	3 000	40	0.914	1890	150	1820	31.2
1	22.5	1.002	1470	6 000	40	0.859	2520	280	2070	24.2
1	1.8	1.000	1410	12 000	40	0.793	3200	490	2410	11

In the first four columns of Table 3 we find quantities which characterize the initial state of the substance (prior to compression), and in the following four columns we find the state of the substance after compression. Then follows the propagation velocity of the shock wave in uncompressed water, $D = u_1$, and the velocity acquired by water during compression, $u = u_1 - u_2$ (see notations in Chapter 8).

The last column of the Table contains the quantity that characterizes dissipation processes and the damping of the shock wave in water. The quantity T'_{1s} represents the initial temperature which is necessary so that by means of isentropic compression from p_1 to p_2 one can reach the state p_2, T_2, v_2 shown in the Table. The difference between T'_{1s} and T_1, represents the increased temperature reached on account of the irreversible processes in the shock wave front. Imagine a shock compression with p_1 to p_2, followed by isentropic expansion to pressure p_1. After the passage of a shock wave of assigned pressure amplitude (p_2, fifth column in the Table) and of the expansion wave following it, the water temperature will rise from T_1 to T'_{1s}.

This increase in water temperature occurred on account of an irreversible consumption of mechanical (kinetic and potential) energy of the shock wave and, consequently, is directly connected with the damping of the wave. The ratio $\frac{T'_{1s} - T_1}{T_2 - T_1}$ can be used as a standard for this damping.

We can readily see that also in this practical case the general relations are satisfied; wave propagation velocity is greater than sonic velocity in unperturbed water, $D > c_1$; wave propagation velocity with respect to compressed water is smaller than sound velocity in compressed water, $D - u < c_2$.

Let us now look into some formal properties of Hugoniot's adiabatic curve.

Quite interesting and significant is the fact that the Hugoniot adiabatic equation cannot written in the form

$$f(p_1, \varrho_1) = f(p_2, \varrho_2). \qquad \text{(IX-6)}$$

In this respect Hugoniot's adiabatic curve appears to differ from such simple curves as the isothermal or Poisson's adiabatic curve. The equation of the latter is

$$S = S(p, \varrho) = \text{const}, \qquad \text{(IX-7)}$$

which, for instance, for an ideal gas yields

$$S = c_p \ln p - c_v \ln \varrho + \text{const}; \quad p\varrho^{-\varkappa} = \text{const} \cdot e^{-\frac{S}{c_v}}. \qquad \text{(IX-8)}$$

To exhaust all the Poisson curves it suffices to go through the one-dimensional series of value for entropy S. But to exhaust all of Hugoniot's adiabatic curves we must plot an "infinity squared" of curves that correspond to every possible value of p_1 and ρ_1.

That Hugoniot's adiabatic equation cannot be represented in the form $f(p, \rho) = \text{const}$ can be seen from the fact that by compressing, for example, a diatomic gas two times by two shock waves, one of which propagates along the other, we can increase density up to 36-fold, whereas in the case of a single compression density cannot be increased more than 6-fold. Thus, double or, generally speaking, multiple compression by shock waves leads to a state that cannot be attained by single compression. However, in the case of isentropic compression, final pressure fully determines the final density of the substance, no matter how many stages were needed to reach the given final pressure, which follows from the possibility of representing Poisson's adiabatic curve in the form (IX-6).

In the p, ρ plane or in the p, v plane, Poission's adiabatic curve is a curve in which all the points are equivalent. None of the points is a singular point. With Hugoniot's adiabatic curve, however, this is not the case. The initial point ρ_1, p_1 (or v_1, p_1) is a singular point of Hugoniot's adiabatic curve. The nature of this singularity will be determined in the next Chapter by studying the neighborhood of point p_1, v_1 that describes the initial state of the substance prior to compression.

We write Hugoniot's adiabatic equation

$$p = H(\varrho; p_1, \varrho_1). \qquad \text{(IX-9)}$$

From the symmetry of the conservation equations from which Hugoniot's adiabatic equation was derived, it follows that if

$$p_2 = H(\rho_2;\ p_1,\ \rho_1), \qquad \text{(IX-10)}$$

then, conversely,

$$p_1 = H(\rho_1;\ p_2,\ \rho_2) \qquad \text{(IX-11)}$$

(see Fig. 28).

Chapter 10

The History of the Shock Wave Problem

The equation of the connection between pressure and density in a substance subjected to the action of a shock wave, which was derived from elementary considerations and from the study of the conservations laws, led to an unexpected result, namely, the increase in entropy with compression of the ideal gas in a shock wave. Entropy increase follows directly from juxtaposing the initial and final state of the substance, which are associated with one another by the conservation equations. We did not investigate the processes that occurred between the control surfaces A and B (Fig. 23b) which led to entropy increase. Formally, as already mentioned, only the conservation equations are symmetrical with respect to ρ_1, p_1 and ρ_2, p_2. We could also satisfy the conservation equations by investigating the inverse motion, viz., a expansion wave in which expansion occurs within a small interval AB (which we shall not investigate closer) in accordance with the Hugoniot equation. In actual fact, however, such a motion is impossible since entropy would drop in it (this is the so-called Zemplen theorem [99] mentioned earlier). This particular feature of the result of Chapter 9 where, without considering dissipation processes, we came to a change in entropy, creates specific difficulties in the understanding of the theory of shock waves which can be overcome only if we observe the processes inside the region of the change of state proper (between the control surfaces A and B, (Fig. 23b). This has held up considerably the evolution of the theory of shock waves.

It is remarkable that the first three most important works on the theory of shock waves were produced at different time periods but, apparently, completely independently from one another. We shall therefore investigate them not in their chronological order.

Riemann [81] set up for the first time two equations, one for the conservation of matter and one for the conservation of momentum. As a third equation he took Poisson's equation, i.e., he preassigns the conservation of entropy in a shock wave, similarly to the conservation of entropy in non-shock waves in which the effect of dissipation forces, viscosity and thermal conduction, is not considered. The relation between pressure and density obtained by him is pretty close to the real one, and so is the general picture of motion which he discovered. However, Riemann's equations do not fully satisfy the law of energy conservation. Hence we have to regard them as erroneous.

It is interesting that in the 1925 edition of the well-known book "Partial Differential Equations in Mathematical Physics", compiled by Weber on the basis of Riemann's lectures [97], even after the problem had been entirely clarified, he (Weber) expresses peculiar doubts as to whether or not Riemann's equations may still hold when considering turbulence.

The conclusion by Hugoniot [56], with whose name Eq. (VIII-7) is usually associated, has been dealt with in the preceding Chapter.

We shall now take a look at Rankine's book [78], which is most interesting from the viewpoint of physical gas dynamics because the author has a deep understanding of the phenomena occurring in a shock wave.

Rankine examines a motion which could propagate ad infinitum without changing its form, i.e., he studies a turbulence that propagates steadily in a gas. He establishes two control planes (like we did when deriving Hugoniot's adiabatic curve) and sets up the law of conservation of matter and the law of conservation of momentum. Rankine studies a substance which has thermal conductivity but no viscosity. He formulates principles of self-modelling which are of the utmost importance for shock waves. Specifically, he emphasizes that numerically the coefficient of thermal conductivity of a substance may be infinitesimal, but we may not neglect it in a shock wave because the width of a shock wave as well as the magnitude of the gradients are not pre-assigned. The smaller the coefficient of thermal conductivity, the greater we may expect the gradients to be in a shock wave, so that the product of the temperature gradient times the coefficient of thermal conductivity (equal to the amount of heat transferred by thermal conductivity in a unit of time) can remain finite as the coefficient itself approaches zero. This makes us thoroughly understand when we can ignore dissipation forces, in particular thermal conductivity, which is when the magnitude of the gradients is pre-assigned by the equations of motion without thermal conductivity. It also makes us thoroughly understand why we cannot ignore thermal conductivity when the magnitude of the gradient is not pre-assinged or predetermined. An example of the first case is a expansion wave for which we have plotted a solution assuming the absence of thermal conductivity. We found that the width of a expansion wave is of the same order as the distance covered by turbulance. The width of a expansion wave increases linearly in time, and in order of magnitude is equal to

$$\Delta x = \frac{\Delta p}{p} ct.$$

If we take this to be the first approximation since in the plotting of the expansion wave thermal conductivity and viscosity were not considered, and if we want to consider in the following approximation the effect of thermal conductivity and viscosity on the temperature and velocity fields found in the first approximation, then we will see that all the gradients will rapidly grow so small that thermal conductivity and viscosity will have virtually no effect on the result. This, however, is not the case in a shock wave. Should we take as a first approximation an infinitely steep discontinuity, obtained when thermal conductivity and visconsity are equal to zero, then in the next approximation, introducing thermal conductivity and viscosity, we obtain infinite heat flow and an infinitely great increase in entropy. In the case of a shock wave where the equations of motion without thermal conductivity and viscosity do not give any specific value for wave width, the gradients and the wave width connected with them can only be obtained from the consideration of dissipative forces. The width turns out to be precisely such that it gives the increase in entropy required by the conservation equations. Conversely, if in a expansion wave with a finite width commensurable with the dimensions of the system we could disregard the effect of dissipative forces, then in a shock wave, in order that dissipative forces could give a finite increase in entropy, it is necessary that the width of the shock wave should be very small as compared with the dimensions of the system. Owing to this we can disregard dissipative forces everywhere except on the surface of shock waves. These relations have been well explained by Rankine qualitatively for the particular case when the only dissipative factor is the thermal conductivity of the substance.

Rankine's further explanations suffer from excessive complexity. He does set up the energy equation quite correctly, but in the general case of an arbitary substance he does not express intrinsic energy in an explicit form as a function of pressure and density. Instead he uses general thermodynamic formulas which include entropy.

On the processes of heat transfer within the discontinuity, he imposes a condition, $\int T dS = 0$, the physical significance of this condition is that in a shock wave there occurs only an exchange of heat between neighboring layers, so that the amount of heat removed from one layer is equal to the amount of heat received by the other one, which means that there are no exterior heat sources.

It takes Rankine some effort to derive a system of equations equivalent to that in Chapter 8 from the combination with the general thermodynamic formulas, and he then writes the equations for an ideal gas. Thus, Hugoniot's adiabatic equation in its customary form (Eq. (VIII-10)), could be derived from the formulas contained in Rankine's work by means of elementary algebraic transformations. Let us remind the reader, however, that Rankine preceded Hugoniot's work by some fifteen years.

Rayleigh summarized in 1910 the evolution of the history of shock waves [79]. He particularly emphasizes the unfairness involved in the term "Hugoniot's adiabatic curve".

Among the occasional papers it is interesting to note that as early as 1858 the English priest Earnshaw [49] came quite close to creating a theory of shock waves. Like Riemann he proceeded from the investigation of a compression wave of finite width in which (see Chapter 2) the wave crest overtakes the region of low pressure thus resulting in a discontinuity. However, the Reverend Earnshaw all of a sudden makes the surprising inference that nature does not suffer discontinuities or jumps. He makes some obscure statements on reflections, and implies that nature will somehow manage to prevent the formation of a shock wave or of a discontinuity. This is an educational example of the bad influence exerted by an erroneous philosphy on scientific research.

In a latter time, already after the discoveries of Riemann, Rankine and Hugoniot, the French scientist Pierre Duhem (one of the leaders of the "energetics" movement fashionable at the beginning of the twentieth century) denied the existence of shock waves on the assumption that in equations of gas dynamics involving viscosity and thermal conductivity there can be no strict discontinuity [46, 47]. Emile Jouguet, a pupil of Duhem, followed Rankine and pointed out that dissipation forces result in an exceedingly small width. If one disregards it, then one can speak of a discontinuity or a shock wave. Not only did Jouguet clarify Duhem's error, but he greatly contributed to an advance in the theory of shock waves and detonation waves [58, 59, 60]. Yet, to this day French authors, probably on account of Duhem's remarks, frequently speak of "quasi-waves", with a view on the finite width of the front.

Essentially we are dealing here with the general problem of the value and significance of approximate methods or approximate solutions in physics (see the remarkable paper by V. A. Foch [29]). This involves also the question as to when as approximate realization of some formulas or relations justifies the creation of new qualitative concepts.

Rankine also touches upon the problem of expansion waves, and refers to an oral communication by Thomson according to which an expansion wave must be mechanically unsteady. In point of fact, however, Rankine already implies the impossibility of a expansion wave (and not its unsteadiness or instability). In fact, if we study the processes of thermal conductivity inside the wave then, besides the conservation equation written by Rankine, $\int TdS = 0$, which states that in a process of thermal conductivity the amount of heat received by one layer is equal to the amount of heat released by other layers, we must take account, at least qualitatively, of the elementary fact according to which in the process of thermal conductivity heat always passes from a hotter body to a cooler one. Hence, of course, we get that in a shock wave entropy can only increase. Thus, were we to type to plot a expansion wave by inverting in a shock wave all the velocities, then inside the shock wave front, inside the "discontinuity" we would also run into the necessity of inverting the heat flow and achieve a transfer of heat from cooler gas layers to hotter ones — which is impossible. We cannot but regret that these elementary considerations are sometimes ignored even in the contemporary literature (see Chapter 1 of Vlasov's book [3], which is otherwise quite valuable).

Chapter 11

Graphical Methods of Shock Wave Theory. Waves Near a Critical Point

A very convenient aid for a simple investigation of shock wave theory is the representation of processes and states on a diagram in which specific volume v is shown on the abscissa and pressure p is shown on the ordinate. We have already mentioned that an assigned initial point (point A, p_1, v_1, in Fig. 27) corresponds to one specific Hugoniot curve. Figure 27 shown how to find on a diagram the propagation velocity of a shock wave. We use a formula which gave us the shock wave velocity as a function of pressure and specific volumes before and after compression

$$D^2 = u_1^2 = v_1^2 \frac{p_2 - p_1}{v_1 - v_2}. \tag{XI-1}$$

For the given initial state of the substance p_1, v_1 the factor preceding the fraction v_1^2 is a constant quantity, and the propagation velocities of shock waves corresponding to various compression stages, various final states, etc., depend on the ratio $p_2 - p_1/v_2 - v_1$, i.e., on the tangent of the dip angle of the corresponding straight lines connecting the initial point p_1, v_1 with the points representing the state of the substance after compression p_2, v_2. Thus, it is obvious from the diagram, that point C where pressure is greater than at point B, corresponds to a shock wave that propagates at a greater velocity because the angle of inclination of the straight line AC is greater than that of the straight line AB. It is quite important that Eq. (XI-1) was derived by us only from the first two equations, the equation of conservation of matter and the equation of conservation of momentum, independently of the equation of energy conservation. Hence it will hold true in all cases where the equation of momentum conservation is not disrupted, i.e., when there is no interference from external forces of the kind of gas friction against walls. In all these cases and in particular also in the presence of a chemical reaction or in the presence of external heat sources or energy sources, which change only the energy equation but not the momentum equation, the relation between density and pressure in the initial and final state, and the propagation velocity remains in force. In particular Eq. (XI-1) refers also to the speed of propagation of a detonation in explosive gas mixture [8, 59, 60].

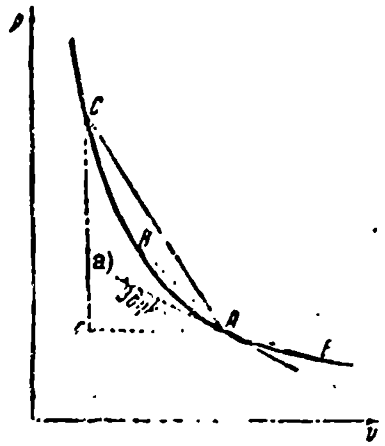

Fig. 27. Shock wave propagation velocity is determined by the slope of the chord, for example, AC, AB, AE. Sound velocity is determined by the slope of the tangent.

CODE: a) Sound.

Special interest should be placed on the fact that Eq. (XI-1) is obtained from compiling the equations of the conservation of matter and momentum only for the initial and final state of the gas in the wave. Use of the straight lines AC or AB for computing the velocity does not mean that it has been assumed that the intermediate states (see Fig. 23b) are represented by points on these lines.

Should we be interested in the intermediate state through which passes the compression inside a thin shock wave front, or inside the front of a detonation wave, or any other wave that propagates steadily in a gas, then, along with the external forces which may violate the law of momentum conservation, we must also consider the possible effects of internal forces of gas viscosity which are omitted in the juxtaposition of the initial and final states. If for any reason we can disregard the effect of viscosity or the effect of internal friction, Eq. (XI-1) can be applied to all those intermediate states through which the substance passes on its way from its initial state to its final state. This is precisely the situation in a detonation wave where the wave width depends on the velocity of the chemical reaction and, generally speaking, is quite considerable, so that the effect of viscosity is small. A detailed discussion of the problem and a complete bibliography can be found in [8, 103].

In Fig. 27 we can also readily find the graphic representation of sound velocity. Sound propagation is obtained as an extreme case of the propagation of very weak shock waves. Thus, sound propagation velocity on the diagram in Fig. 27 is given by the extreme inclination of the secant, when the second point representing the final state of the substance, approaches the first point to an infinitely small distance, i. e., it will be given by the inclination of the tangent to Hugoniot's adiabatic curve at the point representing the initial state of the substance under investigation.

We juxtapose Eq. (XI-1) for $p_2 - p_1$ and the expression for sound velocity $c^2 = -v^2 \frac{\partial p}{\partial v}\Big|_S$, and conclude that at the initial point Hugoniot's adiabatic curve touches the line of constant entropy (Poisson's adiabatic curve).

We can see that for an ideal gas with constant thermal capacity, for which Hugoniot's adiabatic curve has the form shown in Fig. 27, the shock wave propagation velocity is greater than sound velocity in the initial gas $D = u_1 > c_1$. By increasing ad infinitum the shock wave pressure we can, in the limit, obtain an arbitrarily great shock wave propagation velocity. Conversely, for a expansion wave in which the final state E in Fig. 27 lies below the initial state, the propagation velocity would be smaller than sound velocity. If at the final state of the compressed gas in a shock wave, for instance, at point B we plot Poisson's adiabatic curve or Hugoniot's adiabatic curve touching the latter at this point, we can likewise determine the relation between shock wave velocity and sound velocity in a compressed gas. For wave propagation velocity with respect to compressed gas we have

$$u_2^2 = (D - u)^2 = v_2^2 \frac{p_1 - p_2}{v_2 - v_1}, \qquad \text{(XI-2)}$$

an expression which is completely symmetrical with the expression for wave velocity with respect to the initial gas. In Fig. 28 we have plotted through B Hugoniot's adiabatic curve H_B, for which state B has been taken as the initial one. According to the symmetry of equations, if B is on H_A, then H_B passes through point A (see Eqs. (IX-10, 11)).[23] At point B, curve H_B touches Poisson's adiabatic curve. From the position of lines H_B and straight line BA in Fig. 28 it follows that $c_1 > u_1 = D - u$, and sound velocity in a gas compressed by the wave exceeds the wave velocity with respect to the compressed gas.

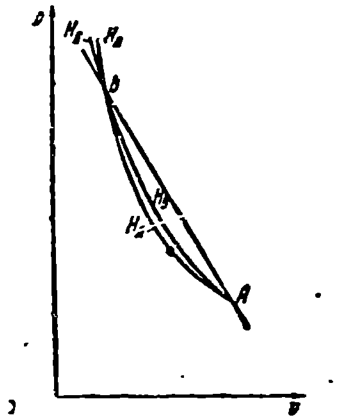

Fig. 28. The relation between wave propagation velocity with respect to initial state A and sound velocity in state A is given by the relation of the inclination of the chord AB and the tangent to curve H_A at point A. The relation of wave velocity with respect to the compressed substance in state B and sound velocity in state B is given by the relation of the inclination of AB and the tangent to curve H_B at point B. A direct comparison of velocities with respect to the various states is inadmissible since the coefficient introduced in the transition from inclination to velocity depends upon specific volume v

In the pv diagram we can analyze the problem of entropy increase in a shock wave. We correlate the expression for the change in intrinsic gas energy in a shock wave with the general thermodynamic expression for energy differential. In a shockwave

$$\Delta E = E_2 - E_1' = \frac{p_1 + p_2}{2}(v_1 - v_2). \quad \text{(XI-3)}[24]$$

But in the general form dE = T dS - p dv. Along Poisson's adiabatic curve (the isentropic line) we would have with a change in volume within the same limits

$$\begin{cases} dS = 0 \\ E_2' - E_1 = \Delta' E = - \int_{v_1}^{v_2} p\,dv. \end{cases} \quad \text{(XI-4)}$$

We correlate the expression for the change of energy along Poisson's adiabatic curve (P) with the expression for the change of energy for shock compression along Hugoniot's adiabatic curve (H) and obtain the equation for the quantity ΔS of entropy change for shock compression

$$\bar{T}\Delta S = \frac{p_1 + p_2}{2}(v_1 - v_2) - \int_{v_2}^{v_1} p\,dv. \quad \text{(XI-5)}$$

Integrals (XI-4) and (XI-5) are taken along Poisson's adiabatic curve.

We investigate the relation between two terms of the last formula in Fig. 29.

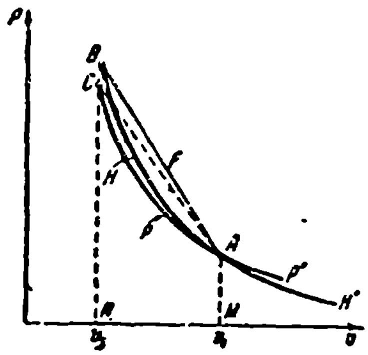

Fig. 29. Entropy increase with compression in a shock wave AB depends on the sign and the size of area AFBCPA. AHB is Hugoniot's adiabatic curve, and APC is Poisson's adiabatic curve.

In this Figure, APC is Poisson's adiabatic curve (the isentropic line), AHB is Hugoniot's adiabatic curve, the change in entropy during compression by the shock wave is $S_B - S_A = S_B - S_C$ and, according to Eq. (XI-5), it depends upon the difference between the area of the trapezium AFBNM and the area limited by Poisson's adiabatic curve APCNM. The product of absolute temperature[25] times entropy increase is equal to the difference between these areas, i. e., the area of APCBF.

We divide this area into two parts by the straight line AC. The first part is a segment the extreme points of which A and C are enclosed by segment APC of Poisson's adiabatic curve chord AC. The second part is the triangle ABC.

We write the equation in the following form, and denote by F the area of the Figures

$$\overline{T}\Delta S = F_{\text{segm. } APC} + F_{\text{tri. } ABC}. \qquad \text{(XI-6)}$$

The area of the trinagle is easy to find. If segment BC is the base of the triangle, then its height is $v_1 - v_2$. The length of BC in the p, v plane is $\left(\frac{\partial p}{\partial S}\right)_v \Delta S$, and the area of the trinagle is

$$\frac{1}{2}\left(\frac{\partial p}{\partial S}\right)_v (v_1 - v_2)\,\Delta S.$$

By substituting this into the initial equation, we find

$$\overline{T}\Delta S = \text{Segm.} + \frac{1}{2}\left(\frac{\partial p}{\partial S}\right)_v (v_1 - v_2)\,\Delta S, \qquad \text{(XI-7)}$$

$$\Delta S = \frac{\overline{\text{Segm.}}}{\overline{T} - a}, \qquad \text{(XI-8)}$$

where $a = \frac{1}{2}\left(\frac{\partial p}{\partial S}\right)_v (v_1 - v_2)$.

In the case of slight volume changes, virtually $\overline{T}\Delta S = F_{\text{segm.}}$, and the correction for the triangle area is small. If $\Delta S \sim (v_1 - v_2)^n$, then the triangle area $\sim \Delta S (v_1 - v_2) \sim \sim (v_1 - v_2)^{n+1}$ is of higher infinitesimal order as compared with ΔS and, consequently, also of a higher infinitesimal order as compared with the area of the segment.

It follows that the sign of the change of entropy is fully determined by the sign of the segment area, i. e., by the reciprocal position of Poisson's adiabatic curve and its secant, which, in turn, depends on the convexity or concavity of Poisson's adiabatic curve, that is, on the sign of the second derivative $\left(\frac{\partial^2 p}{\partial v^2}\right)_s$. If α approaches T, then ΔS approaches infinity, which actually takes place in an ideal gas with $v_2 \to \frac{k-1}{k+1} v_1$, when on Hugoniot's adiabatic curve p approaches infinity. $\overline{T} < \alpha$ corresponds to negative pressure and similar conditions which in the given case are devoid of any physical significance.

For weak waves we can now easily find the extreme laws of entropy change in a shock wave. We expand all the expressions in a power series $v = v - v_1$ and leave everywhere only the senior term which gives us a final result different from zero.

Poisson's adiabatic equation is

$$p = p_1 + \left(\frac{\partial p}{\partial v}\right)_{S,1} \cdot \Delta v + \left(\frac{1}{2}\frac{\partial^2 p}{\partial v^2}\right)_{S,1} \cdot (\Delta v)^2. \qquad \text{(XI-9)}$$

The second subscript shows that the values of the derivatives are taken at state 1 (point A, Fig. 29).

We write $\Delta v_2 = v_2 - v_1 = \omega$, find[26] pressure p'_2 at point C (Fig. 29), omit the subscript of the derivatives and get

$$p_2' = p_1 + \frac{\partial p}{\partial v}\omega + \frac{1}{2}\frac{\partial^2 p}{\partial v^2}\omega^2. \qquad \text{(XI-10)}$$

We write the expression for entropy change, disregarding the area of triangle ABC in Eqs. (XI-5, XI-6, and XI-7)

$$\overline{T}\Delta S = \frac{p_1 + p_2'}{2}(-\omega) - \int_0^\omega p\, d\Delta v = -\frac{1}{12}\left(\frac{\partial^2 p}{\partial v^2}\right)_s \omega^3 = $$

$$= \frac{1}{12}\frac{\partial^2 p}{\partial v^2}(v_1 - v_2)^3. \qquad \text{(XI-11)}$$

By correlating Hugoniot's adiabatic equation in the form

$$I_2 - I_1 = \left(\frac{v_2 + v_1}{2}\right)(p_2 - p_1) \qquad \text{(XI-12)}$$

with the expression DI = T dS + v dp, we can interchange p and v in all the preceding considerations. Thus we get[27]

$$\overline{T}\Delta S = \frac{1}{12}\frac{\partial^2 v}{\partial p^2}(p_2 - p_1)^3 = \frac{1}{12}\frac{\partial^2 p}{\partial v^2}\left(-\frac{\partial p}{\partial v}\right)^3 (p_2 - p_1)^3. \qquad \text{(XI-13)}$$

In a weak shock wave, entropy change is proportional to the cube of the amplitude. At the initial point Hugoniot's adiabatic curve touches Poisson's adiabatic curve. At that point these curves have a mutual tangent and a mutual curvature center (second order tangency). Tangency is accompanied by intersection (see continuation of curves for $v > v_1$ in Fig. 29).

Jouguet [58] obtained these results for the first time without resorting to the simpler, geometric treatment. Since Jouguet's more complete work was published before Zemplen's communication [99] (in the second note in the 142nd volume, Zemplen remarks that he should have quoted Jouguet) the generally accepted custom of calling the proof of the impossibility of discontinuous expansion waves the "Zemplen theorem" is totally incorrect and unfair.

In studying Eq. (XI-11) we establish that for an ideal gas Poisson's adiabatic curve is everywhere convex[28] toward the abscissa. This leads us to the conclusion that entropy increases in a compression shock wave. Conversely, in a sharp expansion wave to which the conservation equations were applicable, entropy would drop, ehence we immediately see that in an ideal gas the propagation of a expansion wave with a thin front, similar to a compression shock wave, is impossible.

For weak waves, Fig. 29 makes it possible in a completely general form, that is, for an arbitrary equation of state of the substance, to conclude that there is a relation between shock wave propagation velocity and sound velocity in the substance before and after compression. For compression to propagate in a gas in the form of a shock wave with an extremely steep front, it is necessary that Poisson's adiabatic curve by convex downward, i.e., have the form shown in Fig. 29. In this case, however, it is geometrically obvious that the inclination of the tangent toward the adiabatic curve at point A must be smaller than the inclination of the secant AB. Conversely, the slope of the tangent at point B, which represents the final state, or the slope of the tangent at point C (which is extremely close to B) is greater than the slope of the secant.[29] Thus, we obtain the elementary conclusion of the relation found for the first time by Jouguet, according to which compression propagates in the form of a shock wave if sound velocity before compression is smaller than the propagation velocity of the shock wave found from the conservation laws, and sound velocity in the substance after compression is greater than shock wave velocity with respect to the compressed substance.[30] In the case of Poisson's adiabatic curve, which has an inverted concavity (Fig. 30, section AB), compression in the shock wave would be accompanied by a drop in entropy since the area bounded by Poisson's adiabatic curve is greater than the area bounded by the secant, the verticals and the abscissa. In a substance in which Poisson's adiabatic curves have an inverted sign of concavity, the compression waves will not be stronger. For instance, a compression caused in any portion of the substance by the movement of a piston propagates in the form of a wave that gradually expands like the expansion waves in an ideal gas discussed earlier. Conversely, in such a substance an expansion wave propagates with an extremely steep front, the steepness of which does not decrease in time and is determined by the small values of thermal conductivity and viscosity. This corresponds to an inverse relation ratio of between shock wave velocity and to sound velocity. In fact in an expansion wave in which the original state is represented by point A, and the final state by point B (Fig. 30), the propagation velocity AB with respect to the substance at state A is determined by the slope of the straight line AB and exceeds sound velocity at state A. This can be seen from the nature of the intersection of the adiabatic curve and the secant at point A, where the tangent to Poisson's adiabatic curve has a flatter slope that straight line AB. Conversely, at point B, that describes the state of the substance after the passage of a steep expansion wave the speed of sound exceeds the speed of propagation of the final disturbance.

Are there such substances in nature in which, at least in some portion of the p, v plane, Poisson's adiabatic curves have a convexity directed upward? We may expect to find such a state near the critical point of a fluid, which is a gas. In fact, long before that critical point, the isothermal curves have an inflection (at the very critical point, the inflection of an isothermal curve becomes horizontal); for a substance with a sufficiently high molecular heat, in which the isothermal and adiabatic curves differ but slightly, we can expect that outside the region of biphase systems, in a state in which the substance is steadily in one phase, the adiabatic curve will also have an inverted sign of the second derivative. The relation between the structure of a compression wave and an expansion wave will become inverted as compared with the relation between a sharply outlined compression shock wave and a blurred expansion wave in conventional gases far away from the critical point.

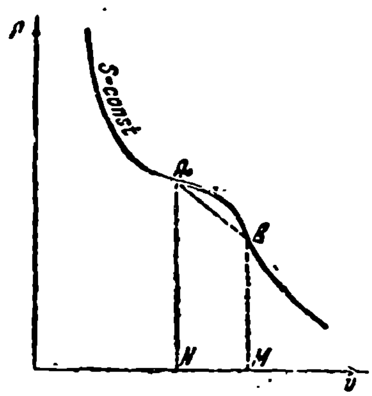

Fig. 30. Poisson's adiabatic curve with an anomalous convexity directed upward. In that section, expansion shock waves are possible.

In Fig. 31, in the plane p, v, for the case c_v = 40 cal/degree mole, we have plotted line II that divides the region with $\left(\frac{\partial^2 p}{\partial v^2}\right)_s < 0$, the adiabatic curve passing through this region, and line I that divides the hatched area of biphase systems (the latter does not depend on the quantity c_v). Computations have been performed with the aid of F. Ye. Yudin (Combustion Laboratory IKhF.)

In the Van der Vaals equation, thermal capacity with constant volume depends only on temperature in the entire region of monophase systems; the energy of a homogeneous substance given by the Van der Vaals equation, can be written in the form of a sum of two terms

$$E = E_1(T) + E_2(v) = \int c_v \, dT - \frac{a}{v}.$$

This considerably facilitates computations since the entropy of a Van der Vaals gas can also be represented in the form of the sum of the temperature function and the specific volume function. It would be very interesting to study experimentally the shock waves and expansion waves in a gas with great thermal capacity in the region where we may expect the existence of the aforementioned anomalies.

For this purpose one can take a high-molecular organic compound that does not decompose at critical temperatures.

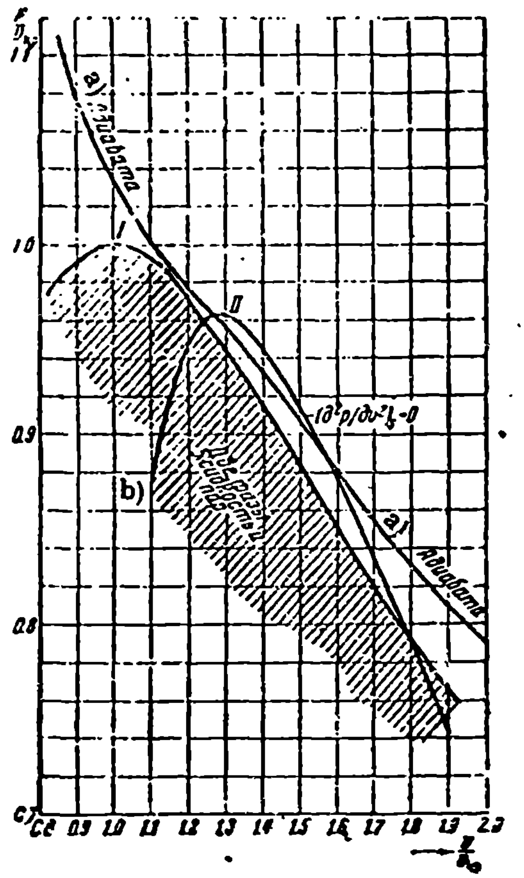

Fig. 31. Adiabatic curves with anomalous convexity in a Van der Vaals gas with thermal capacity c_s = 40. The hatched area represents the biphase systems; curve II limits this area of anomalous convexities. Below curve II $\left(\frac{d^2 p}{dv^2}\right)_s < 0$.

CODE: a) Adiabatic curve, b) biphase fluids and vapor.

The establishment in a general form of a relation between sound velocity in a substance before and after the passage of shock wave, and the change of entropy in a shock wave is quite satisfying since it is obvious (see Thomson's remark quoted in Rankine's paper [78]) that the relation between shock wave velocity and sound velocity determines the mechanical steadiness of the wave. It is essential that the shock wave propagate at a velocity exceeding sonic velocity in the gas subjected to its effect, in order that the disturbance caused by the shock wave does not precede it at a velocity equal to that of sound. It is also essential that the shock wave propagate with respect to compressed gas at a velocity less than sonic velocity in the compressed gas, because only in this case can we imagine a causative relation between the motion of a piston producing a shock wave and the propagation of the shock wave since the disturbance is transferred from the piston to the shock wave front across a layer of compressed gas. The same criteria $c_1 < u_1$, $c_2 > u_2$ will be encountered when studying the onset of shock waves. It is very significant that these perceptible criteria of the mechanical steadiness of a shock wave can be strictly associated with the sign of entropy change in a shock wave. In general terms this determines the possibility or impossibility of the propagation of a shock wave that satisfies the laws of the conservation of matter, the conservation of momentum and energy.

The relation between the sign of ΔS and the inequalities regarding sound velocities will be violated only in the case where in the pressure change interval under study there occur both signs of $\partial^2 p/\partial v^2$, so that Poisson's adiabatic curve has more than two points of intersection with the straight line. Study of the complex conditions under which there occur simulatneously discontinuities and dissipated waves adjoining them exceeds the scope of the present monograph.

Chapter 12

Structure of Shock Wave Front

We shall now investigate the thin layer of a shock wave inside which there occurs the transition from one state to another, i. e., the layer between control surfaces A and B in Fig. 23b. We have not yet discussed the processes taking place inside that layer because its thickness, determined by dissipation forces, is extremely small and the results of the processes occurring there can be determined from the conservation equations without a thorough study of the processes proper.

Here, however, we are specifically interested in the processes occurring inside the layer, and also the thickness of that layer. We shall study separately two extreme cases: Case 1, of extremely small viscosity, and case 2, of very small thermal conductivity. The mathematically (but not physically) more complex case of a simultaneous effect of viscosity and thermal conductivity will not be investigated. For it we will give only the final expression for the thickness of the transition layer.

The first case is remarkable in that Eq. (XI-1)

$$D^2 = v_1^2 \frac{p - p_1}{v_1 - v},$$

which combines the change of density, the change of pressure and wave propagation velocity, turns out to be applicable not only for the final state attained during compression, but also to all the intermediate states within the layer.

As a matter of fact, this equation is the consequence of the first two conservation equations, namely, the conservation of matter and momentum.

The equation of conservation of matter in a simple form (VIII-1)

$$\varrho u = \frac{u}{v} = \text{const}$$

is always satisfied for the propagation of a plane wave. When the wave travels in a pipe, the cross-section of the pipe must be constant. Moreover, the pipe walls must not absorb or eliminate matter. To satisfy the momentum equation for the initial and final state in the simple form (VIII-2)

$$p + \varrho u^2 = \text{const}$$

the substance must not be affected by external forces. During propagation in the pipe, we must disregard friction against the pipe walls. Finally, in studying the intermediate states interesting us here, Eq. (VIII-2) can only be satisfied if the forces of internal friction (viscosity) are small.

In a shock wave travelling through a medium in which there only occur processes considered by the energy equation, for instance, energy release from a chemical reaction (detonation wave, see [8, 59, 60]) or thermal conductivity, Eq. (XI-1) can be applied to all intermediate states. Taking the propagation of a shock wave as a whole, the speed at which each intermediate state moves with respect to the initial state is identical. In Eq. (XI-1), the quantity D must be considered constant. Thus, this equation leads to a linear relationship between pressure and volume

$$p = p_1 + \frac{D^2}{v_1} - \frac{D}{v_1^2} v. \qquad \text{(XII-1)}$$

In the p, v plane (Fig. 32) the state changes along the straight line that connects the points describing the initial (A) and the final (B) states of the substance.

If we know the relation between pressure and density that is valid for the entire shock wave front, we can find its width by means of elementary integration.

It can be shown that along straight line AB entropy attains maximum somewhere in the middle (point M, Fig. 32) between the initial and the final states of the substance.

As a matter of fact, at point A the speed of the wave with respect to the substance is greater than sound velocity, and at point B the speed of the wave is less than sonic speed. At some point M wave velocity equals sound velocity. At this point the straight line AB touches Poisson's adiabatic curve, and, consequently, entropy is maximal.[31]

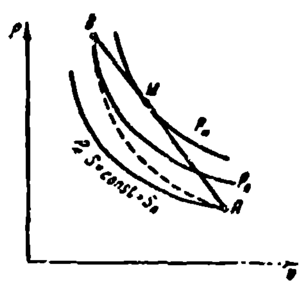

Fig. 32. A and B are the initial and the final state of the gas compressed by a shock wave. The solid lines are Poisson's adiabatic curves, i.e., lines of constant entropy that increases from S_A to S_B and S_M. In the absence of viscosity, but in the presence of thermal conductivity, the state changes along straight line AB on which entropy attains maximum at point M. In the absence of thermal conductivity, but in the presence of viscosity, the state changes along the dashed curve AB, on which entropy monotonically increases from A to B. Hugoniot's adiabatic curve is not plotted here (it also runs through A and B but does not coincide with the dashed line).

If we assume that there is no viscosity, entropy changes only on account of thermal conductivity. Under steady-state conditions, in a system of coordinates in which the shock wave itself is at rest, we can readily change from the substantial derivative with respect to time to the derivative with respect to the coordinate. In this case the sign of the partial derivative is superfluous since the process under study is stationary in the system selected, and does not depend on time. Finally,

$$\varrho T u \frac{dS}{dx} = \frac{d}{dx} \lambda \frac{dT}{dx} = \lambda \frac{d^2T}{dx^2}, \qquad \text{(XII-2)}$$

where λ is thermal conductivity of the substance. The temperature, at least in a weak shock wave, changes monotonically along straight line AB.

The solution sought, the distribution of temperature and entropy as functions of the coordinate, takes the form shown in Fig. 33. The point at which entropy attains maximum values coincides exactly with the inflection point of the dependence of temperature on the coordinate.

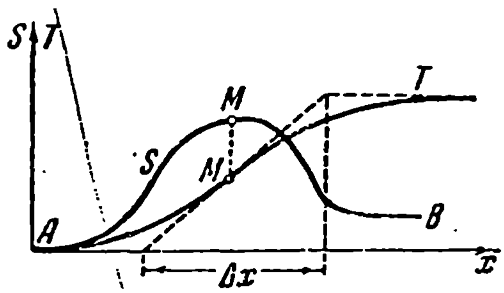

Fig. 33. Internal structure of a shock wave of small amplitude in the presence of thermal conductivity, but without viscosity. Notations are the same as in Fig. 32.

From the preceding chapter we can readily find the order of magnitude (considering the change in volume during compression a quantity of first order of smallness) Δp, ΔT first order, proportional to Δv; $S_x - S_A \sim S_x - S_B$, second order, proportional to $(\Delta v)^2$, $S_B - S_A$ — third order, proportional to $(\Delta v)^3$. It is easy to evaluate the width of the shock wave front by integrating (XII-2) up to point M

$$\frac{1}{v} Tu(S_x - S_A) = \lambda \left(\frac{dT}{dx}\right)_y \simeq \lambda \frac{\Delta T}{\Delta x}. \tag{XII-3}$$

For our evaluations it follows that

$$\Delta x \sim \lambda \frac{\Delta T}{S_x - S_A} \sim \lambda \frac{\Delta v}{(\Delta v)^2} \sim \frac{\lambda}{\Delta v}. \tag{XII-3a}$$

For the determination of Δx, in accordance with the last formulas, see Fig. 33. We establish the order of magnitude of the coefficient from

$$\Delta x \simeq \frac{\lambda}{R} \frac{v^2}{c \Delta v}, \tag{XII-4}$$

where R, the gas constant, is in cal/degree x gram, v and c are selected to give the length.

Figure 33 is a concrete representation of Rankine's ideas [78].

It is interesting to note that in the case of very strong compression, there arises a rather peculiar fundamental difficulty, which consists in that on line AB between points A and B maximum temperature is attained only if pressure in the shock wave P_B exceeds 1.5 P_A (with $c_p/c_v = 7/5$ for a diatomic gas). Maximum temperature is reached at higher pressures than maximum entropy.

In the presence of maximum temperature it turns out to be impossible to plot a continuous distribution of temperature and entropy in space that could satisfy the fundamental equation (XII-1).

Rayleigh [79] has shown that because of this difficulty it becomes necessary to consider also viscosity. However, the effect of molecular viscosity changes not only the energy equation, but also the equation of motion (our Eq. (VIII-2)). Thus, in this case the line of the system in the p, v plane deviates from line AB. Becker [38] made the same considerations at a later date, but without mentioning Rayleigh (referring, however, to a private communication by Prandtl, see also [76]).

In the second extreme case, in the absence of thermal conductivity and the effect of viscosity alone, entropy in the wave changes only on account of the conversion into heat of work performed against viscosity (see Eq. (I-18)).

$$\varrho T \frac{dS}{dt} \sim \eta \left(\frac{\partial u}{\partial x}\right)^2. \tag{XII-5}$$

According to this last equation, entropy under the effect of viscosity increases monotonically. The change in state on the p, v diagram is shown by a curve enclosed between Poisson's adiabatic curves which pass through the initial and final points (dashed line in Fig. 32). We introduce again the concept of effective width

$$\frac{\partial u}{\partial x} = \frac{u_B - u_A}{\Delta x}, \tag{XII-6}$$

$$\frac{dS}{dt} = D\frac{\partial S}{\partial x} = D\frac{S_B - S_A}{\Delta x}. \tag{XII-7}$$

From Eq. (XII-5) we readily find (we identify D and c by their order of magnitude)

$$\Delta x \simeq \frac{\eta v^2}{\Delta v c}. \tag{XII-8}$$

and note that $u_B - u_A = D \cdot \Delta v/v$,

Deviation from straight line AB occurs because of viscosity momentum. The equation of steady motion for one coordinate is

$$\varrho u \frac{du}{dx} = -\frac{dp}{dx} - \frac{d}{dx}\left(\frac{2}{3}\eta \frac{du}{dx}\right). \tag{XII-9}$$

We integrate and find[32]

$$p + \varrho u^2 + \frac{2}{3}\eta \frac{du}{dx} = p_A + \varrho_A u_A^2 = p_B + \varrho_B u_B^2, \tag{XII-10}$$

but from the continuity equation we find

$$u\rho = \frac{u}{v} = M = \text{const}; \quad \frac{du}{dx} = M\frac{dv}{dx}, \tag{XII-11}$$

$$p + Mv + \frac{2}{3}\eta M\frac{dv}{dx} = p_A + Mv_A = p_B + Mv_B = \text{const.} \tag{XII-12}$$

without the term $\frac{2}{3}\eta\ M\frac{dv}{dx}$, the equation yields the straight line AB.

If in accordance with Fig. 32 $\left(\frac{\partial^2 p}{\partial v^2}\right)_s > 0$, then the dashed line enclosed between adiabatic curve $S = S_A$ and $S = S_B$, runs entirely below the straight line, so that in the wave

$$p + Mv < p_A + Mv_A. \tag{XII-13}$$

In this case from the equation we find $\eta M\frac{dv}{dx} < 0$, v in the wave decreases, and compression occurs. An expansion wave would require negative viscosity. By investigating the structure of the wave front under the effect of viscosity we came to the same conclusions regarding the possibility of compression or expansion waves with the sign $\left(\frac{\partial^2 p}{\partial v^2}\right)_s$, which we reached earlier by following another method.

In the case of complete absence of thermal conductivity, a decrease in the viscosity factor leads only to a decrease in the front width, so that there is an increase in derivative du/dx, $\eta\frac{du}{dx}$ remains constant, and the line in the p, v plane does not change.

With thermal conductivity the decrease in front width and the increase in the derivatives with respect to x with decreasing viscosity will be limited. With a sufficiently small η, the entire term η du/dx will be small, and we approach the satisfaction p + Mv = const, i. e., the equation of straight line AB (see, incidentally, our earlier remarks concerning strong shock waves in which on a segment of straight line AB maximum temperature occurs. In this case in a specific portion of the wave front it is viscosity, no matter how small it may be, that determines the magntude of the derivatives).

To evaluate the order of magnitude of front width, we use the molecular-kinetic expression for the coefficient of thermal conductivity and the viscosity factor. In both extreme cases we readily find

$$\Delta x \sim l\frac{v}{\Delta v} \sim l\frac{p}{\Delta p} \sim l\frac{a}{a_1 - a_2}, \qquad \text{(XII-14)}$$

where l is the length of the free path of the molecules in the gas.[33]

For air at atmospheric pressure, taking the Pandtl number (the ratio of kinematic viscosity to thermal diffusivity) to be equal to 1, Taylor [93, 24] with the aid of diffusion coefficient B gives the following expression for the width of the shock wave front

$$\Delta x = \frac{4.4B}{a_1 - a_2}. \qquad \text{(XII-15)}$$

For air at atmospheric pressure B = 0.18 cm^2/sec,

$$\Delta x = \frac{1}{a_1 - a_2} = 4.10^{-5}\frac{1}{\Delta p}\ (\Delta x - \text{см},\ u - \text{см/sec},\ \Delta_p - \text{атм}). \qquad \text{(XII-16)}$$

All the estimates unanimously indicate that in shock waves in which $\Delta v \cong v$ and $\Delta p \sim p$ the width of the front is of the order of the length of the free path. Under such conditions, detailed computations of the structure and the application of the differential equations of hydrodynamics become meaningless.

Chapter 13

Propagation of Shock Waves in a Gas with Delayed Excitation of Internal Degree of Freedom

In Chapter 2 we investigated the propagation of sound in a gas with delayed excitation of the internal degree of freedom, i.e., in a gas in which thermal capacity, with extremely rapid changes in state, is considerably less than with slow changes in state or with slow changes in temperature. This dependence of thermal capacity on the rate of the change of state, this delayed excitation of thermal capacity may be due either to a difficult transfer of energy to the internal degree of freedom, or to a reversible chemical reaction. In thermodynamics, additional thermal capacity due to a reversible chemical reaction whose equilibrium shifts with changes in temperature or pressure, is equivalent to a delayed excitation of the internal degrees of freedom. Conversely, the case of a reversible chemical reaction has nothing whatsoever in common with the irreversible flow of a chemical reaction in a shock wave, i.e., with the phenomenon of detonation, which will not be discussed here.

As mentioned in Chapter 2, the delayed excitation of a portion of thermal capacity leads to two fundamental peculiarities of the acoustic behavior of a substance. First, it leads to sound dispersion, i.e., to the dependence of sound velocity on frequency. High-frequency sound propagates as if thermal capacity were small. In low-frequency sound with a long wavelength, the state changes very slowly. Thermal capacity has time to be fully excited and, consequently, sound velocity is decreased. Simultaneously with sound dispersion, there may occur an exceedingly powerful sound absorption. As one researcher once said, in a specific frequency range the gas develops an "opacity" to sound. There occurs absorption due to the fact that the intrinsic energy of the gas does not change in phase with its pressure or specific volume, i.e., it changes all the time in a state which is far from being in equilibrium, it changes irreversibly. The delayed excitation of a portion of thermal capacity is one of the possible mechansims of dissipation (dispersion) of energy.

We investigate the propagation of a shock wave in a gas with a delayed excitation of a portion of thermal capacity. In the p, v plane (Fig. 34), we can trace through a given point $A(p_0 v_0)$ which describes the initial state of the substance prior to compression, two Poisson adiabatic curves, i.e., two insentropic curves, one of which occurs with an extremely rapid compression (dashed line, $\omega = \infty$), and the other, steeper one, occurs with slow compression and full excitation of the entire equilibrium thermal capacity of the substance (dashed line, $\omega = 0$). If we are interested in the propagation of shock waves over long distances (we shall see later what is the natural scale of this problem and with respect to what distance may be considered great), the control plane on which we fix the state of the gas subjected to compression, can always be set at a sufficient distance from the spot where compression began, so that there will always exist a region in which all internal degree of freedom and the entire intrinsic thermal capacity are fully excited. As we place the control plane, Fig. 23b, in that spot, we obtain from the conservation equation a Hugoniot adiabatic curve with full excitation of the internal degrees of freedom (solid line AMC, $\omega = 0$). Consequently, this curve at point A touches at point A the flat Poisson curve which corresponds to low frequency, and only farther, at considerable compression values, moves away from it and runs steeper.

It can be seen from Fig. 34 there can be different cases depending on pressure from compression in a shock wave. A weak shock 1 (in which the final state after compression, after complete excitation of all the internal degree of freedom, is described by point M on Hugoniot's adiabatic curve, $\omega = 0$) must propagate with a velocity that is less than sound velocity at high frequencies. Which will be the structure of such a shock wave.

If in the comparatively weak shock wave 1 under study there occurred in some section of the front an extremely rapid and abrupt change in state, then to this change we could also apply the conservation laws. However, in the case of a rapid change of state, the excitation of external degrees of freedom has no time to occur. Such a change of state may be called "a shock wave without excitation".

A Hugoniot adiabatic curve plotted without consideration of the internal degrees of freedom, i. e., for an extremely rapid compression, must lie higher than the corresponding Poisson's adiabatic curve (the solid line AB, $\omega = \infty$ in Fig. 34). The propagation velocity of this "shock wave without excitation" is obviously greater than sound velocity at a high frequency, and, consequently, it exceeds all the more low-frequency sound velocity, and even exceeds the velocity of sufficiently weak shock waves with excitation.

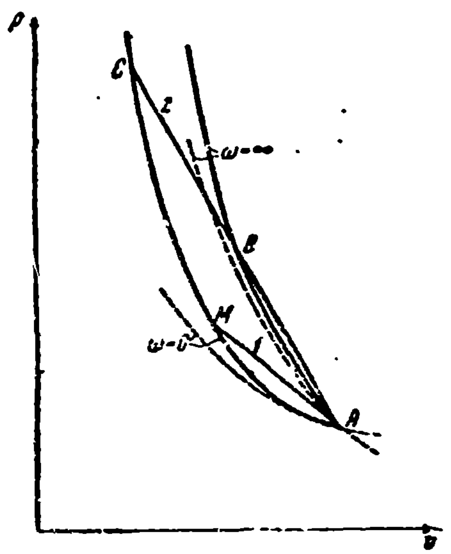

Fig. 34. Propagation of a shock wave in a gas with delayed excitation of a part of thermal capacity. Hugoniot's adiabatic curve (solid line) and Poisson's adiabatic curves (dashed line) are plotted on the basis of two assumptions, the absence of excitation ($\omega = \infty$) of a portion of thermal capacity, and total excitation. The chord of Hugoniot's adiabatic curve $\omega = 0$ intersects or does not intersect the adiabatic curve $\omega = \infty$ depending on amplitude.

Thus, in the mode sought for, in order that it be stationary (if it is stationary), if all the parts of the front move at the same velocity with respect to the gas and conserve the distance with respect to one another at a constant front structure, there can be no abrupt pressure increase or abrupt changes in volume in a weak wave. We may say that from a slowly propagating disturbance, a slowly moving shock wave, there will continuously emanate high-frequency sound waves the velocity of which will exceed the velocity of the shock wave owing to sound dispersion. These waves, however, dampen very quickly, indeed they dampen exponentially ahead of the shock wave. The accumulation of an infinite amount of damping sound waves forms a "washed-out" front of a weak shock wave. We can find the exact structure of the front by ignoring in this case the effect of viscosity and thermal conductivity. The state of the substance changes along straight line AM. The rate of this change depends on the excitation rate of the internal degrees of freedom. It is qualitatively obvious (and it can be corroborated by fairly simple computations) that the effective front width of such a shock wave which propagates at a velocity less than the velocity of high-frequency sound, depends on the excitation rate of the internal degrees of freedom. In order of magnitude, the front width is equal to the product of sound velocity times the excitation rate of thermal capacity (see Fig. 35a). This width may exceed many times the width of the front obtained from the effect of viscosity and thermal conductivity. Thus, in the case of carbon dioxide, total thermal capacity with slow excitation amounts to 3.3. cal/mole x degree, of which 2.5 cal/mol x degree represents the thermal capacity of rotational and progressive molecule motion, excited instantly, virtually with every collision between molecules. The remaining 0.8 cal/mole x degree is oscillatory thermal capacity excited, as an average, once every 600,000 collisons [62]. At high frequencies, sound velocity exceeds by 4% sonic velocity at low frequencies. In carbon dioxide, a shock wave caused by the motion of a piston at a velocity of approximately 13/sec, in which a compression by 5% is attained (pressure increases by 7%), propagates in the gas at a velocity which is still 1% less than high-frequency sound velocity. By computing from Prandtl's [76], Rayleigh's [79], Taylor's [93] and Becker's [38] formulas (Chapter 12) the width of such a shock wave in air, where it depends on thermal conductivity and viscosity, we get at atmospheric pressure 8×10^{-3} mm, and 0.4 mm at a pressure 15 mm Hg. In carbon dioxide these values would be even smaller. However, the width of a shock wave in carbon dioxide, where it depends on delayed excitation, amounts according to a rough computation to 12 mm at atmospheric pressure. At a pressure of 150 mm Hg, the width reaches 60 mm. Such a sharp change in the width of the shock wave can be noticed when studying the front structure by means of Topler photography when comparing photographs in gases, such as air, in which there are no such effects, and photographs in carbon dioxide.

In the case of a strong shock wave (2 in Fig. 34) we must expect more complex modes (see Fig. 35b). Discontinuity AB, the width of which is determined by vicosity and thermal conductivity, and is therefore extremely small, propagates without a noticeable excitation of the internal degrees of freedom, point B lies on the corresponding Hugoniot adiabatic

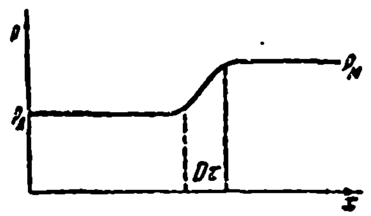

Fig. 35a. Structure of a shock wave of small amplitude (AM) in a gas with delayed excitation (see Fig. 34). τ is excitation time.

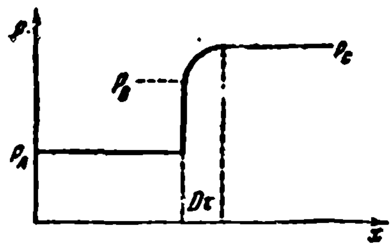

Fig. 35b. Structure of shock wave of great amplitude (AC) in a gas with delayed excitation (see Fig. 34).

curve (sign $\omega = \infty$). The excitation behind the discontinuity is accompanied by a smooth (in length of the order $D\tau$) increase in pressure and compression up to point C. Figures 35a and 35b show the distribution of pressure in the shock wave front which may be expected in these two cases. The distributions of temperature, density and velocity, not shown here, are quite similar. The photographic study of the form of a shock wave must, we feel, become an expedient direct method of investigating delayed excitation of internal degree of freedom.

The increase in the front width becomes a natural phenomenon if we remind ourselves of the fact that delayed excitation yields a large second viscosity factor (Chapter 1). However, substitution of actual concepts by the formal introduction of the second viscosity factor is possible only in a limited extent and, in particular, does not permit the finding of the more complex mode in Fig. 35b (see Chapter 2, $\omega \gg 1/\tau$). Detailed computations can be found in a paper by this author to be published in Zhurn. eksper. teor. fiziki (Journal of Experimental and Theoretical Physics).

Chapter 14

Formation of Shock Waves

We discussed the theory of shock waves proceeding from the motion resulting from the compression of a gas by a piston that at a specific instant ($t = 0$) begins to move to a constant velocity. We arrived at a mode at which the shock wave is formed as soon as the piston begins to move, and propagates with constant intensity. With a finite piston mass, such a motion would require that an inertia of infinite magnitude be overcome at the initial instant with an instantaneous change in piston velocity.

Let us study the motion of a gas caused by the gradual accleration of a piston compressing the gas, which is at rest when motion begins. We can easily plot this motion by substituting continuous acceleration by a large number of minimal velocity jumps, i.e., by substituting the smooth curve in the $x - t$ plane by a broken line consisting of chords of that curve.[34]

We thoroughly investigate the first stages of this motion. The piston begins to move, and it moves during a time t_1 at a small constant velocity w_1.

During that time a shock wave of constant intensity propagates in the gas. The velocity of the substance affected by the shock wave is constant and equals the velocity of the piston w_1. In other words, the piston is at rest with respect to the gas immediately adhering to it. The same is repeated until the next velocity jump to w_2 takes place, and a second shock wave, characterized by the velocity jump $w_2 - w_1$ travels in the gas adjoining the piston and compressed by the first shock wave, etc.

Figure 36 shows velocity distribution in space after three such jumps. The distribution curves for pressure and density at the same instant have an analogous shape.

A fundamental significance is acquired now by the properties of shock waves, shown in their general outlines by Jouguet ([58, 60], see also Duhem [48]). The propagation velocity of wave 1 with respect to the gas compressed in it in segment 2-1 is smaller than sound velocity at state I.

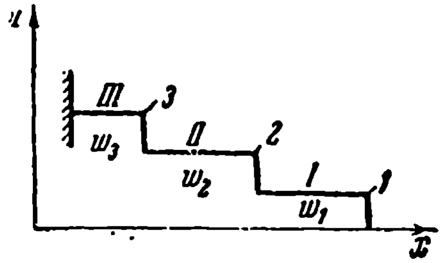

Fig. 36. Propagation of a series of subsequent momenta. In time, point 3 catches point 2, and both points catch point 1. The ordinate shows gas velocity.

Conversely, the velocity of wave 2 with respect to state I, which for this wave is the initial velocity, must be greater than sonic velocity at state I and, according to Jouguet, all the more exceeds the velocity of wave 1.

Hence we see that the waves catch one another, tend to accumulate and combine into a powerful shock wave. Hugoniot attributes to this phenomenon the stability of shock waves [56]. Hadamard [54] and Becker [38] compute the moment and place at which accumulation begins as a function of the acceleration of the piston.

In the x, t plane, accumulation corresponds to the intersection of characteristics (lines representing the motion of individual shock waves) ahead of the piston.

In the case of exhaustion (piston movement away from the gas) the characteristics spread in a fan-like fashion without intersecting, and the solution found (see Chapter 6) remains correct for an unlimited amount of time. By decreasing the individual velocity jumps and by increasing their number we come to a continuous, smooth curve of piston motion and to a continuous distribution of density, pressure, and velocity in the gas ahead of the piston, instead of steps.

In the case of compression, however, such a solution will be correct only until the characteristics intersect, i.e., until such time when one wave catches the preceding one.

As the magnitude of the velocity jump $w_n - w_{n-1}$ decreases and the time interval between two consecutive velocity jumps also decreases proportionally, the time and the place at which two waves join (the intersection point in the x - t plane) approach a fully determined limit. Let us find that limit. The propagation velocity of a very weak waves does not differ from sound velocity. In a gas in motion, to sound velocity there is added the motion of the gas proper which is equal to the speed of the piston, so that the propagation velocity of a weak wave in space is equal to $c + w$. During a time Δt the wave covers a distance $(c+w)\ \Delta t$.

If during that time the speed of the piston has changed by Δw, and the compression caused by the change in piston speed changed sound velocity by Δc, the propagation velocity has increased by $\Delta w + \Delta c$. This is the velocity at which one wave catches the other (the difference between their velocities), so that the waves will meet after a time $t = \frac{c+w}{\Delta c + \Delta w} \Delta t$.

By using the laws of change of state in weak waves applied to acoustics (we could obtain them also by an ultimate transition from the shock wave equations), we can readily compute the latter quantity

$$t = \frac{c+w}{\Delta c + \Delta w} \cdot \Delta t = \frac{c+w}{\frac{\Delta c}{\Delta w} + 1} \cdot \frac{\Delta t}{\Delta w}. \qquad \text{(XIV-1)}$$

After transition to limit we obtain

$$\frac{\Delta t}{\Delta w} = \frac{1}{dw/dt} = \frac{1}{g}, \qquad \text{(XIV-2)}$$

where g is the acceleration of the piston.

$$\frac{\Delta c}{\Delta w} = \frac{dc}{dw} = \frac{dc}{d\rho}\frac{d\rho}{dw}. \qquad \text{(XIV-3)}$$

In acoustics we found that

$$du = \frac{c}{\rho} d\rho; \quad c = \sqrt{\frac{\partial p}{\partial \rho}}.$$

Since gas velocity u is equal to piston velocity w, we obtain

$$\frac{d\rho}{dw} = \frac{\rho}{c}; \quad \frac{dc}{d\rho}\frac{d\rho}{dw} = \frac{\rho}{c}\frac{dc}{d\rho} = \frac{d \ln c}{d \ln \rho}. \qquad \text{(XIV-4)}$$

For an ideal gas we readily find

$$c \sim \sqrt{T} \sim \rho^{\frac{k-1}{2}}; \quad \frac{d \ln c}{d \ln \rho} = \frac{k-1}{2}, \qquad \text{(XIV-5)}$$

$$t = \frac{c+w}{1 + \frac{k-1}{2}} \frac{1}{g} = \frac{1}{g} \frac{2}{k+1} (c+w). \qquad \text{(XIV-6)}$$

In the case of an arbitrary equation of state we transform the denominator in Eq. (XIV-1) in the following fashion

$$\frac{dc}{dw} + 1 = \frac{\rho}{c}\frac{dc}{d\rho} + 1 = \frac{1}{2\rho c^2}\left(2\rho^2 c \frac{dc}{d\rho} + 2\rho c^2\right) =$$

$$= \frac{1}{2\rho c^2} \frac{d}{d\rho} \rho^2 c^2 = \frac{1}{2\rho c^2} \frac{d}{d\rho} \rho^2 \frac{dp}{d\rho}. \qquad \text{(XIV-7)}$$

We switch to a more convenient variable, the specific volume $v = \frac{1}{\varrho}$; $\frac{d}{dv} = \varrho^2 \frac{d}{d\varrho}$, and find

$$\frac{dc}{dw} + 1 = \frac{v^3}{c^3}\left(\frac{\partial^2 p}{\partial v^2}\right)_S \qquad \text{(XIV-8)}$$

and within the limit when $w \to 0$

$$t = \frac{c^3}{v^3 (\partial^2 p/\partial v^2)_S} \frac{1}{g}. \qquad \text{(XIV-9)}$$

Thus, the possibility of one wave to catch the preceding one, and the possibility of a shock wave being formed are connected with the sign of $(\partial^2 p/\partial v^2)_S$, the role of which in thermodynamic theory has already been noted in Chapter 11.

A comprehensive study of all the aspects of motion in the case of arbitrarily assigned piston motion runs into great difficulties [54, 38]. There arise shock waves of a finite but variable amplitude, and after their passage the entropy of the gas changes. Only very recently Kibel, Frankl and Khristianovich succeeded in developing effective graphical computation methods which, however, are much too complicated for our course (see [11]). Analytical methods have hitherto been found only for the motion prior to the formation of a discontinuity [37].

It is obviously much easier to find such a motion of the piston whereby all the characteristics intersect at one point, i. e., all the waves catch one another simultaneously and at one single spot.

Let us assign the place and time of the formation of a shock wave (the conjunction of all the weak waves), which are interconnected by the condition $x_b = c_0 t_b$, found from the study of the first weak wave that propagates in an unperturbed and still motionless gas. We transpose the origin of the coordinates in the x, t plane to that point (and we get new coordinates x', t') and find that the state of the gas is constant along the straight lines (characteristics) which go through the origin of the new system of coordinates. In other words, the state of the gas depends only on the ratio x'/t'.[35] In particular, gas velocity and piston velocity equal to it also depend only on x'/t'.

Thus, the differential equation of piston motion is homogeneous

$$w = \frac{dx'}{dt'} = f\left(\frac{x'}{t'}\right) \qquad \text{(XIV-10)}$$

and can be readily integrated (see Smirnov, Course of Higher Mathematics. Vol. 2, p. 80).

The form of function f can be found by noting that the slope of the characteristic is

$$\frac{x'}{t'} = u + c. \qquad \text{(XIV-11)}$$

The relation between u and c in the case of the change of the gas caused by waves propagating in one direction (see Chapter 6) in the absense of shock waves (with constant entropy) can occasionally be found in its explicit form [ideal gas $u = \frac{2}{k-1}(c - c_0)$]. This relation can always be found for a given adiabatic equation p = p (ρS = const) in a parametric form [u = u (ρ), c = c (ρ)], see Eqs. (VI-10).

We transform it to

$$u = f(u + c), \qquad \text{(XIV-12)}$$

where f is precisely the function f of Eq. (XIV-10).

Thus, for an ideal gas in the case that $k = c_p/c_v = 1.4$ there takes place

$$u = \frac{2}{k-1}(c - c_0) = 5(c - c_0) = \frac{5}{6}(c + u - c_0), \qquad \text{(XIV-13)}$$

$$\frac{dx_a'}{dt'} = \frac{5}{6}\left(\frac{x_a'}{t'} - c_0\right). \qquad \text{(XIV-14)}$$

We introduce the dimensionless parameter y

$$\frac{x_a'}{t'} = yc_0; \quad x_a' = c_0 t' y; \quad \frac{dx_a'}{dt'} = c_0 t' \frac{dy}{dt'} + c_0 y. \qquad \text{(XIV-15)}$$

According to Eq. (XIV-14), we get

$$c_0 t' \frac{dy}{dt'} + c_0 y = \frac{5}{6}(yc_0 - c_0). \qquad \text{(XIV-16)}$$

The variables split up as follows

$$t' \frac{dy}{dt'} = -\frac{1}{6} y - \frac{5}{6}. \qquad \text{(XIV-17)}$$

The initial conditions are

$$t_0' = -t_i; \quad x'_{0a} = -x_i = -c_0 t_i = c_0 t_0'; \quad y_0 = 1. \qquad \text{(XIV-18)}$$

The solution has the following form

$$\left.\begin{aligned} t' &= -t_i\left(\frac{y}{6} + \frac{5}{6}\right)^{-6}, \\ x_a' &= c_0 y t' = -t_i c_0 y\left(\frac{y}{6} + \frac{5}{6}\right)^{-6} = -x_i y\left(\frac{y}{6} + \frac{5}{6}\right)^{-6}. \end{aligned}\right\} \qquad \text{(XIV-19)}$$

We revert to the system of coordinates in which at an initial instant the piston was at the origin of the coordinates, and obtain the following equation for piston motion in its parametric form

$$x_n = x_A\left[1 - y\left(\frac{1}{6}y + \frac{5}{6}\right)^{-6}\right], \tag{XIV-20}$$

$$t = t_A\left[1 - \left(\frac{1}{6}y + \frac{5}{6}\right)^{-6}\right]. \tag{XIV-21}$$

In its explicit form the equation is quite clumsy.

The curve of Eqs. (XIV-20) - (XIV-21) is plotted precisely in Fig. 37. The piston velocities are marked at various points. The dashed line represents the first characteritic.

The amplitude of the discontinuity in density, velocity and pressure at the intersection point depends on the instant at which the motion of the piston deviates from the law just established.[36]

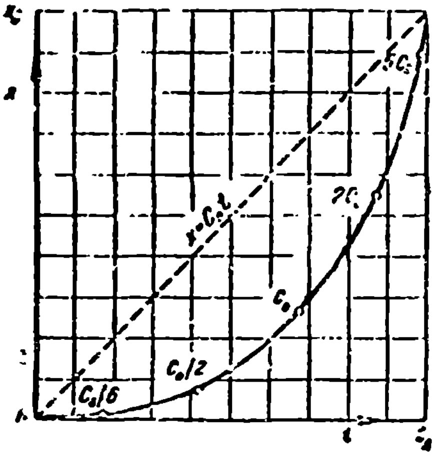

Fig. 37. Piston motion (solid line) for which all the characteristics intersect simultaneously at one point A in the upper right-hand angle of the drawing. Piston velocity is marked at individual points. c_0 is sound velocity in the gas before compression.

A final discontinuity occurs at the intersection point the moment all the waves join. One can readily see, however, that this discontinuity cannot propagate further as one whole without any change, since in a discontinuity propagating without change (a shock wave) there exist other relations between density, pressure and velocity. Thus, until the occurrence of the discontinuity the gradients everywhere were small, the effect of dissipative forces could be disregarded, entropy did not change, and the relation between pressure and density satisfied Poisson's adiabatic equation. In a shock wave, Hugoniot's equation is satisfied, and entropy increases.

The motion that arises when the discontinity occurs will be investigated in Chapter 16. In the next chapter, Chapter 15, we give some experimental data on the occurrence of shock waves.

Chapter 15

Shock Waves in the Case of Oscillations of Large Amplitude

Around 1860 it was noticed that strong electric sparks from a Leyden jar formed strange lines on a smoked plate. The electric origin of these lines was suspected. Mach et al. [66, 67, 68, 69, 82] showed in a series of ingenious experiments that these lines are the trace of a collision of waves which propagate from individual sparks and are reflected at the borders of the plate. By placing at the plate two spark intervals of different lengths connected in series, Mach noted that the point where the waves meet is always closer to the weak spark. Thus he showed the dependence of the propagation velocity of strong disturbances on their amplitude. By using the shadow method for observing the propagation of waves, stroboscopy and instant photography with light from an individual spark, Mach showed the supersonic propagation velocity and the sharpness of the distrubance front. He also noted that a disturbance that propagates in (three-dimensional) space fades out much quicker than a disturbance forced to propagate in one dimension only, such as in a narrow tube.

Vieille around 1900 performed experiments aimed at indicating shock waves that arise in a pipe when a partition dividing gases of different pressure was ruptured [96]. Vautier investigated the propagation of the momentum caused by a shot from a pistol [123]. In the first case, the relation between pressure (wave amplitude) and its propagation velocity that follows from Hugoniot's equations, was proved with sufficient accuracy. In the second case, the waves were relatively weak, and had at their origin a "washed-out" front without a discontinuity. However, over a stretch of several kilometers (Vautier used a recently built but not yet operating water supply line) one could note a gradual, characteristic increase in steepness, and the formation of a discontinuity in the wave front.

We shall now briefly dwell upon the last tests [87, 70] which were conducted with particular care. Gas vibrations were studied in an inlet and exhaust pipe of an internal combustion engine [87] according to the following characteristics A pipe 12 m long with an inner diameter of 7 cm was attached to the cylinder of a small piston engine having the same diameter (7 cm) and a piston travel of 6.8 cm. At five different points inside the pipe devices measuring gas pressure and velocity were placed. Pressure was measured with a piezo-electric crystal, and velocity was measured with a 2 x 3 mm disk attached to the axis of the pipe.

The disk moves along the pipe axis with the motion of the gas, and turns a rod. The rotation of the rod is recorded through a small window with the aid of a mirror attached to the rod. Particular attention was paid to the high proper frequency (low inertness) of the measuring instruments, and the satisfactory damping of proper vibrations.

An electric motor imparted the piston a harmonic alternating motion. The amplitude of the oscillations was small at frequency values far away from resonance values. The change in pressure and velocity in each cross section of the pipe also occurred according to harmonic law, in complete accordance with the conventional concepts of acoustics.

In the case of resonance, however, the type of motion changed abruptly. Figures 38a and 38b show schematically the recordings of the devices in the case of excitation of the fundamental tone of the pipe. Piston oscillation frequency is 14.4 hertz (14.4 oscillations per second). The amplitude of gas motion is extremely wide, as should have been expected. At a frequency of 14.4 hertz, piston velocity does not exceed π 14.4 h, where h is piston travel, i.e., 3.14 x 14.4 x 6.8 cm/sec = 3.1 m/sec. In the case of resonance, gas velocity attains 25 m/sec, that is, about 10 times more. Of particular interest to us is the shape of the curves for the change of velocity and pressure, which evidences the occurrence of shock waves of considerable amplitude in the case of harmonic excitation by a comparatively slow moving piston.

The theory of shock waves permits us to reach approximate though extremely important conclusions regarding the amplitude of waves with resonance under Schmidt's test conditions. Energy dissipation from friction and heat transfer from the gas near the

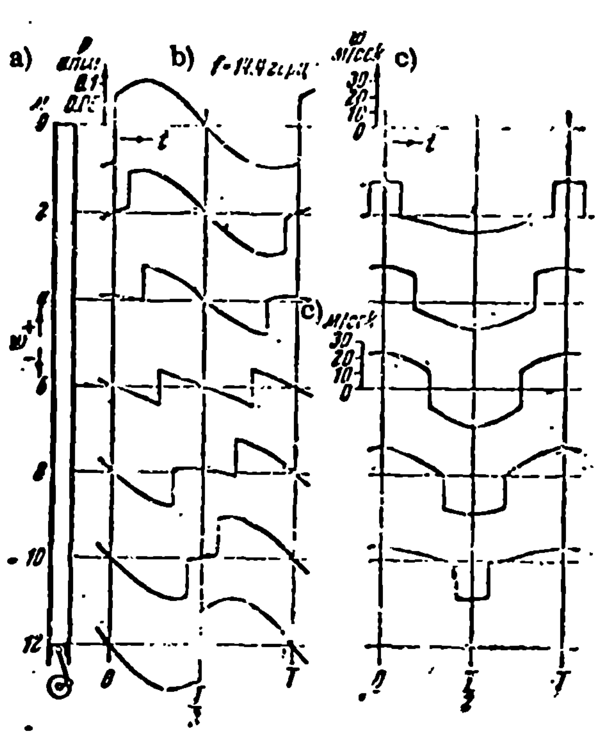

Fig. 38a. Diagram of test (on the extreme left, the pipe) and record of changes in pressure (left) and gas velocity (right) in 7 cross sections of the pipe depending on time with oscillation excitation by piston motion with a fundamental proper frequency of the pipe of 14.4 hertz.

CODE: atm; b) hertz; c) m/sec.

lateral walls of the pipe (Kirchhoff [61]), in the case of reflection from the end of the pipe and the piston (Konstantinov [13]) all these causes for sound absorption, common for acoustics, are very small under conditions of experiments of this type. The amount of energy dissipated during a unit of time grows proportionally with the square of the amplitude (i.e., proportionally with oscillation energy) and at great amplitudes, when discontinuities occur, it may even become secondary as compared with other mechanisms of energy dissipation.

We established in Chapter 11 that in a shock wave there occurs an increase in entropy proportional to the third power of the amplitude of pressure, density or velocity in the wave. Under steady-state conditions, this increase in entropy must be compensated by an automatically occurring transfer of heat from the gas into the pipe walls.

The entropy increase describes the irreversible transformation of mechanical energy into thermal energy; it describes the damping of waves negligible in the case of small amplitudes, and the rapidly increasing (by the cube rather than by the square as in linear acoustics) absorption. We introduce the effective value of pressure amplitude Δp, denote frequency by ω, the pipe length by l, piston travel by h, piston velocity by w, piston area equal to the pipe's cross section by F and we find, approximately, the work performed by the piston during a unit of time

$$A=\frac{1}{t}\int_0^t F\Delta p\, w\, dt. \qquad \text{(XV-1)}$$

In the case of resonance[37] we approximately evaluate A, and note that w approximately equals $h\omega$.

$$A\simeq \Delta p h\,\omega F. \qquad \text{(XV-2)}$$

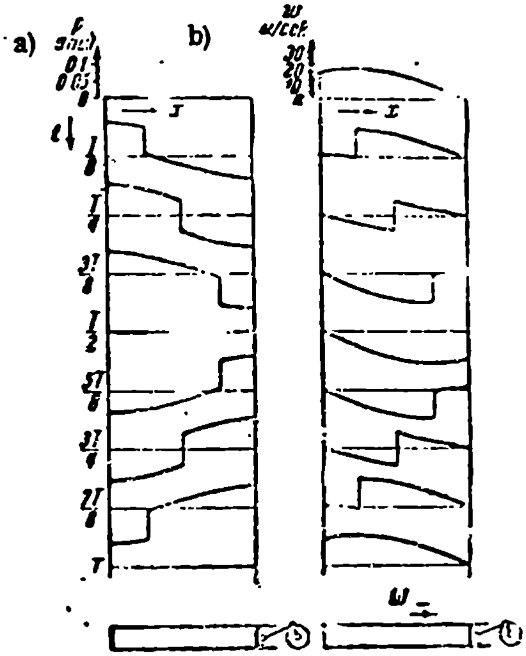

Fig. 38b. Instantaneous distributions of pressure and velocity lengthwise in the pipe at various instants of time (processing of the recordings in Fig. 38a).

CODE: a) atm; b) m/sec.

We find energy absorption by setting up the expression

$$A_1 = D\varrho F T \Delta S, \tag{XV-3}$$

where $D\varrho F$ is the amount of substance subjected to shock compression during a unit of time; D, shock wave propagation rate, is substituted approximately by sound velocity c; ΔS is increase of specific (per gram) entropy; T is absolute temperature; $T\Delta S$ is work per gram of substance irreversibly transformed into heat.

According to Eq. (XI-13),

$$T\Delta S \simeq \frac{1}{12}\frac{\partial^2 v}{\partial p^2}(\Delta p)^3.$$

For air k = 1.4;

$$\left(\frac{\partial^2 v}{\partial p^2}\right)_S = \frac{v}{p^2}\frac{1}{k}\left(\frac{1}{k}+1\right) = \frac{2.4}{1.4^2}\frac{v}{p^2}.$$

$$T\Delta S \simeq \frac{1}{10}\frac{v}{p^2}(\Delta p)^3;$$

$$A_1 = c\varrho F T\Delta S = \frac{1}{10}c\varrho F v(\Delta p)^3/p^2 = \frac{cF}{10}\frac{(\Delta p)^3}{p^2}. \tag{XV-4}$$

We equate the work performed by the piston to energy absorption and obtain

$$\left(\frac{\Delta p}{p}\right)^2 = \frac{10h\omega}{c}.$$

In the case under study of excitation of the pipe's fundamental tone, piston oscillation frequency in resonance is connected with pipe length $\omega = c/2l$ (the length is of the half-wave equal to pipe length). By substitution we find the simple formula

$$\frac{\Delta p}{p} = \sqrt{5\frac{h}{l}}. \tag{XV-5}$$

In Schmidt's experiment, h = 6.8 cm and l = 12 m, we find

$$\frac{\Delta p}{p} = \sqrt{5\frac{0.068}{12}} = 0.17; \quad \Delta p \sim 0.17 \text{ atm abs.}$$

which is in a reasonable relation with the observed order of magnitude (Figs. 38a and 38b) if we take into account the approximate nature of the computation and the existence of other types of absorption. Let us note that for overtones, alongside the change in the relation between ω and l, we must also consider the presence at every instant of several discontinuity surfaces (shock waves), which increases E_1.

Chapter 16

Propagation of an Arbitrary Discontinuity

In Chapter 14 we have come very close to considering the problem of the subsequent fate of a discontinuity that arises at the junction spot of several weak shock waves, a discontinuity that is not governed by Hugoniot's equation. We generalize that problem and formulate the problem of the behavior of an arbitrary discontinuity as follows.

At an initial instant of time $t = 0$ there is given a plane (located at the origin of the coordinate $x = 0$) in which all the quantities p, v, T, u, which characterize the state and the motion of the gas, are subject to a jump. On both sides of the discontinuity plane, all these quantities are constant. The greater the distance at which all of these quantities can still be considered constant, the longer (in terms of duration) will the solution to which we come be correct.

Since the conditions of the problem do not contain either a characteristic length or a characteristic time, analysis of Chapter 6 shows that one must seek a motion that depends only on the relationship x/t. In Chapter 6 this motion was found analytically for the propagation of an expansion wave in a gas. For a compression wave the analytic solution led to an absurd conclusion, namely, to the necessity of realizing at one and the same point in space simultaneously three different values for pressure and volume. Precisely this absurdity became the starting point for the development of shock wave theory. The knowledge of shock wave theory enables us to solve both particular problems for piston motion that begins at the time instant $t = 0$, which leads either to an expansion wave or to a shock wave. Now we can also solve the general problem of the propagation of arbitrary discontinuities. We sill set up the solution from expansion waves and compression waves studied earlier.

Let us first note a specific difficulty. The expansion wave propagates in a gas at a velocity equalling that of sound, whereas the compression wave, as we have seen, propagates at a velocity exceeding that of sound. However, with respect to the already compressed gas, the compressed shock wave propagates at a velocity less than that of sound. Thus, we have only two waves. One wave, either the expansion or the compression wave, propagates in one direction, for instance, to the left of the plane in which occurred the discontinuity at an initial instant of time, and the other wave propagates in the opposite direction, namely, to the right. We cannot direct more than one wave in one direction. In fact, if, for instance, a shock wave propagates to the right, then the expansion wave and, all the more, the shock wave that travels in the gas subject to compression in the same direction, are bound to catch up with the original shock wave. But since both waves must proceed from one point x = 0 simultaneously at the instant t = 0, when the discontinuity occurred (in other words, the entire phenomenon must depend only on the coordinate x/t, and in this case it is inconceivable that one wave catch up with the other), then there can be no more than one wave travelling in one direction. However, a wave that propagates in a gas the state of which is assigned (it may be either a shock wave or an expansion wave) can be fully determined by one parameter. Thus, for example, if we determine the density ratio before and after the passage of a shock wave, the density will determine the pressure of the shock wave (according to Hugoniot's adiabatic curve), the propagation speed of the shock wave, entropy and all the other quantities of the substance subjected to compression. And in order that it be precisely a shock wave with which we deal, it is necessary that the density of the substance exceed its initial density since we are dealing with gases far away from the critical point. Conversely, if we establish that the density of the substance after the passage of the wave be less than its density prior to the passage of the wave, then on the basis of thermodynamic considerations we can immediately conclude that we are dealing here not with a shock wave but with a constantly expanding expansion wave. For an expansion wave the change in density again fully determines the change in pressure in the wave, the gas entropy in the wave does not change, while the velocity of the wave is equal at any point to sound velocity.

Thus, at first sight it would appear that we have only two parameters by which can be selected the change in density in two waves that propagate in two different directions. We need, however, also a third parameter in order to describe the propagation of an arbitrary discontinuity. On one side of the discontinuity, for example, on the right, we were assigned three quantities, namely, pressure, density and velocity in an unperturbed gas. For each wave we have one parameter. There are two waves, which gives us two parameters. We must, however, get to the arbitrarily assigned three quantities which characterize the state of the gas on the left (for instance, pressure, density and velocity on the other side of the discontinuity). Thus we necessarily conclude that there must exist another discontinuity, or another wave. However, that discontinuity or wave must have a peculiar property, namely, the discontinuity in question must not propagate at sonic velocity with respect to the gas. We can imagine such a discontinuity only if pressure and velocity on both sides of the discontinuity are identical. Only in this case there will be no sound waves proceeding from the discontinuity towards both sides. The fact that velocity and pressure are equal, a fact which guarantees the mechanical equilibrium in the discontinuity of a special kind, does not interfere with the fact that on both sides of that discontinuity temperature, density and gas entropy are different. With the aid of such a third discontinuity (a discontinuity of a special kind) it becomes possible to satisfy all the equations, i.e., it becomes possible to find a full solution to the problem of the further fate of an arbitrary discontinuity assigned at an initial instant of time.

Let us first of all assign specific values to the pressure and specific volume of the substance.

In the p, v diagram of Fig. 39, let point A represent the state of the gas left of the discontinuity (pressure p_a), and point B be the state of the gas right of the arbitrary discontinuity (pressure p_b) at the initial instant t = 0. We now follow all the motions which result for different values of velocity with respect to the motion of the substance right and left of the discontinuity plane assigned at the initial instant. Through each point A and B we plot upwards Hugoniot's adiabatic curve along which compression in

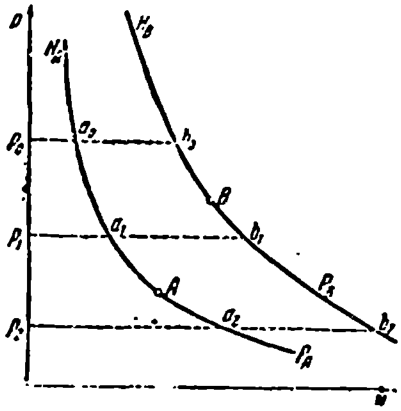

Fig. 39. Propagation of an arbitrary discontinuity. The initial states on both sides of the discontinuity are described by points A and B. Hugoniot's adiabatic curves H_A and H_B are plotted above A and B, Poisson's adiabatic curves P_A and P_B are plotted below.

the shock wave proceeds, and downward we plot Poisson's adiabatic curve along which the state of the substance changes with expansion in the expansion wave.

With a change in relative velocity, there is a change in pressure p in waves propagating in the first and the second gas, the pressure being equal on both sides of the discontinuity of a special kind. However, instead of assigning a relative velocity and finding pressure p, it is more expedient to proceed in a different way, and, establishing various values of p, plot a corresponding regime and determine which should be the relative motion of the gases at the initial instant in the states represented by points A and B in order that the assigned pressure p can be attained.

We select pressure p_0 which exceeds both pressure p_a and p_b (Fig. 40a). In this case expansion waves will travel right and left of the arbitrary discontinuity. The substances in states a_0 and b_0 border on one another. They are divided by the discontinuity of a special kind in which pressure on both sides is equal to p_0 and the velocities of the substance must be equal to one another. But since the substance in

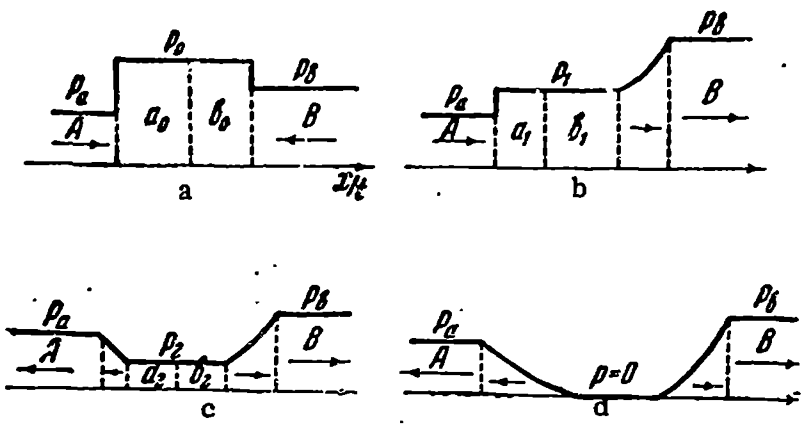

Fig. 40. Characteristic cases of the propagation of an arbitrary discontinuity with assigned pressure and density on both sides of the discontinuity, but different relative velocities ties.

a—collision of two gas masses; there arise two shock waves; b—gas masses moving at different velocities; in the high-pressure gas there arises an expansion wave which pushes the shock wave in the low-pressure gas; c—scattering of two gas masses; there occur two expansion waves; d—scattering of two gas masses at a velocity exceeding the sum of outflow velocities; there occur two expansion waves, with vacuum in in the center.

The arrows showing gas velocities are given in the system of coordinates in which rest the gases occurring in the waves in the center of the diagrams (a_0, b_0, a_1, b_1, a_2, b_2).

state a_0 moves to the left with respect to the initial substance A, and substance b_0, as in the shock wave, moves to the right of its initial substance B, i.e., in the direction of the propagation of the shock wave Bb_0, then, in order that the velocities in states a_0 and b_0 be equal to one another, it is necessary that at the initial state, at the instant $t = 0$, substances A and B move toward each other colliding at high speed. We will obtain shock waves propagating on both sides of the discontinuity in the case of a collision of two masses of substance moving towards one another at high speed. The smaller the velocity at which substances A and B collide, the smaller must be pressure p_0 in the shock waves. Finally, at a sufficiently small collision velocity we go over to another regime (Fig. 40b). In this regime, pressure p_1 is greater than p_a but smaller than pressure p_b. Along substance A a shock wave moves and along substance B an expansion wave moves. Such a regime can be realized, in particular, also if at the initial instant t = 0 the velocities of substance A and substance B are equal to one another, so that at the initial instant we have only a pressure discontinuity. It is obvious that in this case, between substances A and B, there arises an area with a pressure intermediate between p_a and p_b. In this case the substance moves from the higher pressure B toward the lower pressure A. The shock wave travels in the substance in which pressure is lower. Conversely, the expansion wave travels in the substance in which the pressure is higher. This case is examined in detail below.

Let us now return to Fig. 39 and continue the analysis of the various cases that may occur. Selecting p_2 smaller than p_a and p_b, we obtain expansion waves which travel on both sides of the initial discontinuity (Fig. 40c). Such a regime will be realized if at the initial instant the substance in state A and the substance in state B move in different directions from the discontinuity at a sufficient speed. Finally, if and when the relative velocity at which the substance in state A and the substance in state B move away from each other at the initial instant exceeds $5(c_A^0 + c_B^0)$,[38] where c_A^0 and c_B^0 denote sound velocity in state A and in state B, i.e., if the relative velocity of substance A and substance B exceeds the sum of maximum outflow velocities of substance A and substance B, then between substance A and substance B a vacuum will be formed (Fig. 40d).

In a paper by Shchelkin and this author [9], and in an earlier paper by Shardin [84], detailed numerical computations are given that refer to the case of initial pressure discontinuity without velocity discontinuity (the case in Fig. 40b). It is interesting that if the compressed substance is hydrogen, in which sound velocity is greater than in the second substance of low pressure (e.g., air), the shock wave is considerably more powerful than if the compressed substance also were air. Let us take a numerical example from [9]. Figures 41a and b show the distribution of pressure and temperature

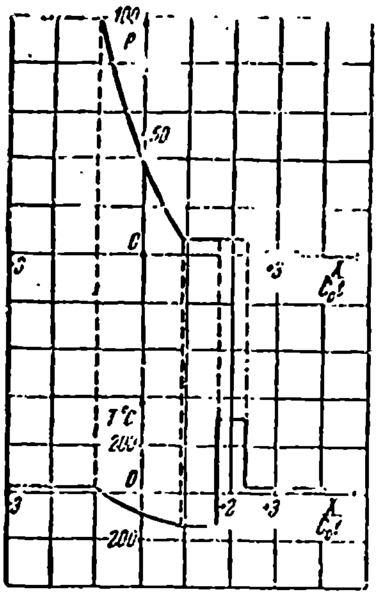

Fig. 41a. Propagation of a discontinuity that arises when air at rest compressed to 100 atmospheres absolute and air at rest at 1 atmosphere absolute touch one another. At the initial instant temperature everywhere is 20°C. The diagram shows the pressure curves (above) and temperature curves (below).

in the case of a sudden rupture of the screen that divides the gas compressed to 100 atmospheres and the gas under atmospheric pressure. The compressed gas in both cases is placed on the left. The abscisas shows the relation between the coordinate and time x/c_0t, where c_0 is sound velocity in the air at initial temperature independent of pressure. The screen was placed at x = 0.

In Fig. 41a (where air is on both sides) we see that on the left at a distance greater than unity the compressed air is still unperturbed. Between $x/t = -c_0$ and $x/t = 0.9\ c_0$ there is an expansion wave which at its last points borders on air expanding to a pressure of about six atmospheres. The discontinuity of a special kind is at rest with respect to the air on both sides of the discontinuity, but in our system of coordinates it moves together with the air surrounding it at a velocity 1.7 times that of sound in the initial

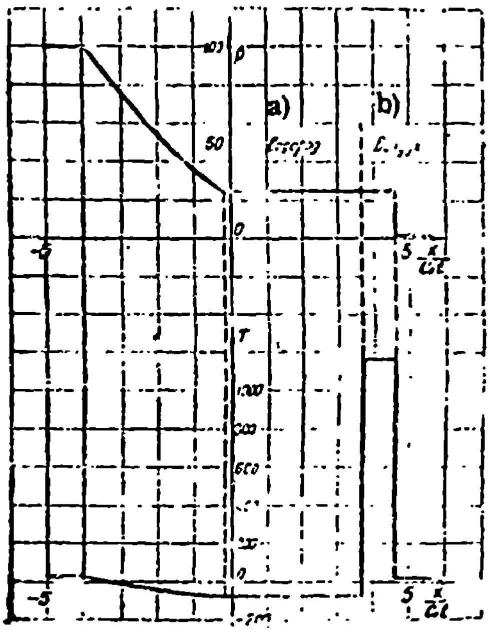

Fig. 41b. Compression of air by hydrogen with an initial pressure of 100 atmospheres absolute.

CODE: a) Hydrogen; b) Air.

stage (i.e., at about 580 m/sec). To the right of the discontinuity of a special kind there is air under shock compression from atmospheric pressure up to a pressure of about 6 atmospheres. In the expansion wave the air temperature drops from 20°C (at 100 atmospheres) to -140°C (at 6 atmospheres) in accordance with Poisson's adiabatic equation. To the right of the shock wave, gas compression from 1 to 6 atmospheres is accompanied by a temperature increase from 20°C to 300°C, which appreciably exceeds the temperature increase according to Poisson's adiabatic equation (220°C). The compression shock wave from 1 to 6 atmospheres propagates at a velocity equal to 2.3 times the speed of sound. Only for x greater than 2.3 c_0t, on the right there is unperturbed air at atmospheric pressure.

Figure 41b shows a similar case where the compressed gas is hydrogen. Because of higher sound velocity, hydrogen is capable of giving a considerably higher outflow velocity for a given pressure differential. Hence hydrogen compresses air considerably more, although hydrogen itself expands much less. Pressure in the expansion wave in hydrogen and in the shock wave propagating in the air amounts to about 25 atmospheres. Accordingly, the shock wave reaches considerably higher velocities, approximately 4.6 c_0.[39] The temperature in the shock wave is extremely high, 1175°C. One may assume that such a high temperature during the outflow of hydrogen into air may, under certain conditions, lead to the ignition of hydrogen. If the outflow of hydrogen into air occurs in a closed container, the subsequent repeated reflection of shock waves may lead to a further increase in temperature.

Which of the cases shown in Figs. 40a, b, c, and d will occur if the discontinuity at the initial instant is formed by the application of a large number of small compression shock waves which simultaneously join at one and the same instant in a point in space? Physically this case can be achieved by pushing into a gas a piston at a variable velocity. In Chapter 13 we found such a curve for the piston motion at which all the waves joined simultaneously. At that instant, on the right of the spot where the waves joined, we have unperturbed gas. On the left we have a gas subjected to repeated compression by weak shock waves.

We have noted several times, however, that the subsequent compression by two shock waves is not equivalent to a one-time shock compression. In particular, entropy increase in each wave, if the waves are sufficiently small, is proprotional to $(\Delta p)^3$. By choosing a sufficiently large number of sufficiently weak shock waves we can achieve compression to any assigned pressure with any small entropy increase, since if we subdivide the entire assigned pressure change between n waves, then pressure increase in each wave is proportional to $1/n$, entropy increase in each wave is proportional to $1/n^3$, and total entropy increase in n waves is proportional to $1/n^2$. Thus, in the case of an accumulation of a large number of weak compression waves we will have a nearly adiabatic change of state.

At the instant of accumulation of individual waves as shown in Fig. 36 (Chapter 13), on the right of the accumulation spot we have an unperturbed gas in the initial state A, and on the left we have a gas in state B which was virtually subjected to adiabatic compression.[40] It is obvious that point B does not lie on Hugoniot's adiabatic curve H_A. Accordingly, the discontinuity cannot propagate further as one whole. We must apply to its propagation the general theory of propagation of arbitrary discontinuities. It can be shown that the velocity acquired by a gas during consecutive compression by a large number of shock waves is smaller than the velocity which the gas would acquire were it compressed to the same pressure by one shock wave. Hence it follows that during propagation of a discontinuity that occurred from the accumulation of many weak shock waves, we will have the case in Fig. 40b. Pressure p_1 will be lower than the pressure produced by the piston (pressure p_B). An expansion wave will travel in the compressed gas in the direction toward the piston, and to the right into the unperturbed gas will travel the compression shock wave created by the discontinuity. Figure 41c shows the distribution of pressure and temperature obtained after a time t following the conjunction of waves formed by the compression of air by a piston the velocity of which gradually reached $4.44\ c_0 = 1500$ m/sec, so that pressure at the piston p_B attained 50 p_A, i.e., 50 atmospheres absolute. Pressure in the compression shock wave will be less than pressure p_B reached earlier at the piston. However, because of entropy increase, this lower pressure corresponds to a higher temperature. Temperature discontinuity in a relatively unperturbed gas is shown in the diagram only for this case (dashed line, Fig. 41c). Let us note that in this figure the coordinate and time are calculated repsectively from the place and the instant of accumulation, i.e., from the occurrence of the discontinuity. In the system of coordinates in which A is motionless, the expansion wave moves to the right; however, it moves to the left with respect to the gas in state B which moves at a great speed, and with respect to the piston, not shown in Fig. 41c.

The case examined above is of considerable interest for the theory of detonations, because the result obtained explains how a flame acting on a gas like a piston can, by gradual compression, produce a shock wave at a great distance from the piston (or the flame). By graudally compresseing the gas to a comparatively low temperature (630°C,

Fig. 41c), we can achieve an abrupt increase in temperature (1450°C, Fig. 41c) at a considerable distance at the instant of accumulation, or achieve a "remote ignition" of the gas. Apparently the mechanism of the occurrence of a detonation in gases must be imagined precisely in this way in a number of cases.

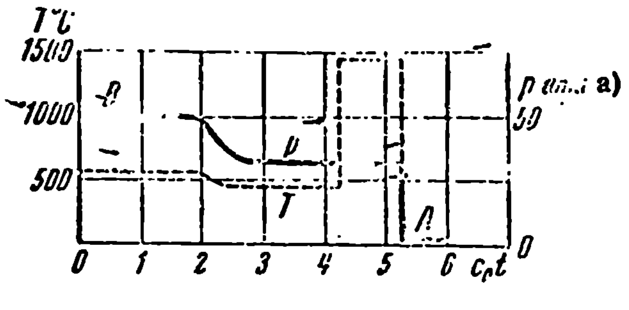

Fig. 41c. Propagation of a discontinuity that occurred after the collision of compression waves in Fig. 36. The pressure in the arising shock wave is lower (the expansion wave moves toward the compression waves), but the temperature in the shock wave is considerably higher than the maximum temperature reached by the accumulation of small compression waves. The solid line is the distribution of pressure, the dashed line represents the distribution of temperature.

CODE: a) p/atm abs.

Having determined the character of the motions obtained during the propagation of an arbitrary discontinuity, we can verify the initial assumption according to which motion depends only on the relation x/t.

In Chapter 6, in the case of an expansion wave, this solution depended on the absence of dimensional values of time or length in the initial and boundary conditions of the problem, and also on the fact that dissipative forces were ignored. The latter is necessary, since in the combination with sound velocity, from viscosity or thermal conduction we can plot the characteristic length and the characteristic time, for example $\frac{\nu}{\rho c_0}$ and $\frac{\lambda}{c_p \rho c_0}$. In an expansion wave, dissipative forces were neglected because the equations of gas dynamics led us to a "wahsed-out" wave of great width (increasing linearly with time), with exceedingly small values of the velocity gradient and the temperature gradient.

Is it possible to neglect dissipative forces in the case of a shock wave in which a considerable entropy increase occurs? A positive answer to that question is associated with the fact that the numerical value of entropy increases in a shock wave (due, in the last analysis, to the effect of viscosity and thermal conductivity) is fully determined by the conservation equations and does not depend on the magnitude of thermal conductivity and viscosity. The latter determine only the final width of the shock wave front. But the dimensional value for length (the width of the shock wave front) thus obtained is extremely small. It is of the order of the length of the path of a molecule in the case of a strong shock wave.

Also small is the width of the discontinuity of a special kind. The equilization of temperature on both sides of this discontinuity, and the mutual penetration of gases by diffusion lead, after a time t, to a width of the order of $\xi = \sqrt{\varkappa t} \simeq \sqrt{Bt}$, where $\varkappa$ is thermal diffusivity; B is the diffusion coefficient. We use the molecular-kinetic expressions $\varkappa$ and B, and find $\xi \sim \sqrt{lct}$, where l is the molecule path length, and c is sound velocity. But the distance x covered by shock waves or expansion waves during a time t, is of the order of ct, so that $\xi \sim \sqrt{lx}$.

Thus, the relationship of the dimensions of the area in which dissipative forces are substantial to the dimension of the entire area covered by motion, is equal to l/x for a shock wave, and $\sqrt{\frac{l}{x}}$ for the discontinuity of a special kind. Both quantities are extremely small in any large-size motion in which x is absolutely greater than l.

Very interesting is the history of the investigation of the propagation of an arbitrary discontinuity, which reflects the different viewpoints of the investigators of various countries characteristic of the study of the theory of shock waves. The above theory had been expounded by Hugoniot as he was formulating the theory of shock waves [56]. Hugoniot's theory of the propagation of an arbitrary discontinuity was well known to other French authors. It is mentioned by Crussard [45], and it is also found in Hadamard's book [54] on the propagation of waves. As a matter of fact, Hadamard's exposition is somewhat distorted by the absence of a clear explanation as to when one should use Hugoniot's adiabatic curve, and when one should resort to Poisson's adiabatic curve (entropy increase in compression shock waves, and the impossibility from a thermodynamical viewpoint of the existence of expansion shock waves were proved later by Jouguet and Zemplen), and also by his attempt to arrive at closed formulas. However, the theory of propagation of an arbitrary discontinuity appears to be unknown to German authors. Thus, Weber [97] discusses only the case of a collision of two shock waves of equal amplitude, i.e., precisely the case when both initial states A and B of our drawing identically coincide and, consequently, all the Hugoniot adiabatic curves plotted from them also coincide. In this particular case, as can be seen from the symmetry, the discontinuity of a special kind becomes zero. On both sides of it not only pressure and velocity are identical, but also temperature, entropy and density are equal to one another. In the 1925 edition of his book, Weber writes that "... it is not yet known what will happen in the general case of a collision of two arbitrary shock waves."

The problem of the accumulation of shock waves was formulated by Becker in his well-known book "On the theory of detonation and shock waves" [38]. In 1920 he correctly predicted the fundamental qualitative result of the accumulation of shock waves, namely, the temperature increase at the instant they coincide. Then he writes: "No one knows yet what will happen when the steepness of the rise will become infinite after a certain time." The solution of this problem is given above. It must be mentioned that in his paper Becker mentions Hugoniot's memoir as well as Hadamard's book. A precise and very general investigation into all the cases of the propagation of an arbitrary discontinuity that may be encountered, is given by Kotchine [64].

Chapter 17

Supersonic Flow Around a Body

Above, in Chapter 4, we clarified some properties of a flow around a body at supersonic speeds, inherent in flows at a great distance from a body. First of all we established a fact, according to which the turbulence caused by the presence of a body in a supersonic flow, involves not the entire flow but only a cone with an axis parallel to the direction of the flow, and the angle of aperture the sine of which is equal to the ratio of sound velocity to flow velocity (this is known as the Mach angle). However, these statements referred only to flows at a great distance from the body. In particular, only at a great distance from a body, where we can regard turbulence to be small, we may state that the turbulence propagation rate will be equal to sound velocity. Close to the body itself, where the turbulence caused by the presence of the body can no longer be regarded as small, this turbulence can steadily spread with respect to the flow in the form of a shock wave at a velocity in excess of that of sound, in an unperturbed gas. The knowledge of shock wave theory makes it possible for us to establish certain properties of flow around a body by a supersonic flow, which refer to the immediate neighborhood of the body flowed around and, which, consequently have a certain importance for the problem of the resistance of a body moving at supersonic speed, which is a problem of paramount importance in modern ballistics.

In the following we will study separately two cases. The first case is the flow around a body with a blunt profile. We can readily imagine the general character of the flow (Fig. 43).

As we have already mentioned, at a great distance from the body, perturbation (or turbulence) is small. The solid line shows the position of the stationary shock wave, the dashed lines represent the flow lines. At a great distance from the body, where the shock wave amplitude is small, its velocity does not differ from sound velocity, and the dip angle of the solid line is equal to the Mach angle. There is no doubt, however, that at some point (and this point can readily be found for any symmetrical profile), the shock

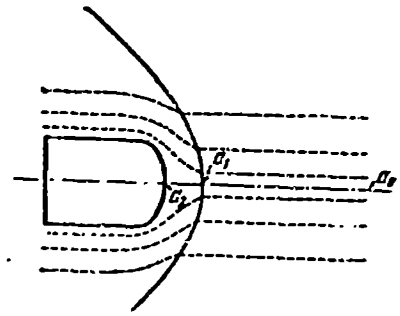

Fig. 42. Diagram of supersonic flow around a body with a blunt profile.

wave surface must run normal to the direction of flow (Fig. 42, point a_1). At that point, gas velocity with respect to the shock wave is maximum, the amplitude of pressure change in the shock wave is greatest and can be readily computed if we know the flow velocity (or, conversely, the velocity of our projectile, or any other body under investigation with respect to the motionless gas). In the case of compression in the shock wave, gas velocity changes from supersonic to subsonic. Thus, in the immediate vicinity of the body, near its blunt front part, we deal with a subsonic flow. Any further slowing-down of the gas on the segment from the shock wave to the body surface, a_1-a_2 (Fig. 42), takes place adiabatically, and pressure increase can be computed with the aid of Bernoulli's theorem.

Rayleigh [79] pointed out a very substantial fact, according to which such a consecutive compression first of the shock wave and then the adiabatic compression in the resulting subsonic flow leads, in the case of high velocities, to a considerably lower pressure than a purely adiabatic (isentropic) compression from supersonic velocity to a state of rest achieved at the point where the flow lines branch off in the front part of the blunt profile being flowed around. The fact that pressure in the case of total slowdown will be lower in the presence of a shock wave can be readily proved thermodynamically. Both in the presence and in the absence of a shock wave, along a flow line, the law of energy conservation holds, i.e., Bernoulli's theorem holds in its integral form

$I + u^2/2 = const$, which fully determines the enthalpy of the gas at the point where it will be "stopped dead", known as enthaply at rest $I_0 = I + u^2/2$. If compression occurs adiabatically, then condition $S = const$ is added. The value of enthalpy I and entropy S fully determine the state of the substance. If a shock wave occurs, then entropy is no longer conserved.

Computation of the exact value of pressure and the computation of the state of the substance ensuing from deceleration in the case where this occurs partially in the shock wave, is far more complex. We can state, however, that entropy in the shock wave increases, and that an entropy increase for a given enthalpy always means a drop in pressure.[41] Thus, the presence of a shock wave ahead of the body moving at a supersonic velocity leads to a decrease in pressure in the front part of the body's blunt profile, leads to a decrease in the resistance to the body's motion, and thus removes (as shown by Rayleigh) the considerable disagreement between experimental data on the resistance of projectiles and the magnitude of resistance as computed by formulas based on adiabatic (isentropic) compression. This is of considerable importance also when measuring supersonic velocities by means of a Pitot tube. In this case it is also necessary to take into account the occurrence of a shock wave in front of the outlet of the tube.

Let us imagine a reservoir filled with a compressed gas that flows out with supersonic velocity, and a body placed into the supersonic flow (Fig. 43). In the reservior the gas is at rest, in the nozzle it gains momentum and, approaching point a_2 on the surface of the body flowed around, it is again slowed down. The comparison between the state of the gas in the reservior and at point a_2 is quite instructive. If the change in the state of the gas during deceleration follows the same law as in the case of acceleration, then at point a_2 the gas should return to the same state in which it was in the reservior, and gas pressure and temperature at point a_2 should not differ from pressure and temperature in the reservoir. This is so in the case of subsonic flow, but in the case of supersonic flow, acceleration and expansion in the nozzle occur isentropically, whereas deceleration and compression of the gas in the shock wave are accompanied by an increase in entropy. We apply the law of energy conservation to the motion of an element of the gas volume and obtain Eq. (III-5), Chapter 3,

$$I+\frac{u^2}{2}=\text{const.} \tag{III-5}$$

This equation holds true, and the value of the constant is maintained, also in the case of shock compression of the gas, i.e., when the flow line intersects the shock wave surface during steady motion.[42] In the reservoir and a point a_2, velocity $u = 0$, hence Eq. (III-5) leads to the conclusion that enthalpy in the gas at the branching point and in the reservoir is the same. Enthalpy in gases depends only on temperature. Hence, in the experiment shown in Fig. 43, the gas in the reservoir cools off during outflow and is heated again during deceleration to reach the same temperature it had in the reservoir (as this has been the case in a subsonic flow). However, the irreversible increase in entropy at the deceleration stage leads to the fact that density and pressure in the gas at point a_2 are lower than in the reservoir, and, unlike in subsonic flows, pressure is not fully restored. This fact is of considerable importance for the resistance of air to the motion of bodies flying at supersonic speeds, and it has been thoroughly investigated by Rayleigh (Table 4).

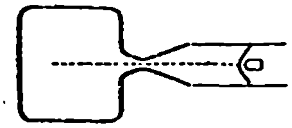

Fig. 43.

Table 4

u, м/сек a)	330	660	990	1320
u/c_0	1	2	3	4
$p(a_2)$, ата b)	1.89	5.75	12.32	21.6
$T(a_2)$, °C	80	250	550	950
$p(S=\text{const})$	1.89	7.84	36.6	150.2
$p(a_1)$	1.00	4.5	10.3	18.5

CODE: a) m/sec; b) atm abs.

The first line of the table gives the velocity of the body (for motion in the air), the second line gives the ratio of body velocity to sound velocity, the third line gives the pressure developed during motion at point a_2, the fourth line gives gas temperature at that point (p_0 = 1 atmosphere absolute, T_0 = 20°), the fifth line gives the pressure that could be developed in the case of isentropic deceleration of the gas or, in other words, the pressure that should have been developed in the reservoir in order to achieve the assigned outflow velocity of the gas. Finally, the last line in the table gives the pressure at point a_1 of Fig. 42, after compression in the shock wave, but prior to deceleration in the subsonic flow.

It is interesting to note that if a body with a blunt profile is flowed around by a gas at subsonic velocity, near the body's surface there may occur an area of supersonic velocity. Thus, if a round cylinder is in a cross flow, supersonic velocity on the side is obtained beginning from the Barstow number Ba-0.45 (Taylor [25]).

In the case of supersonic flow around a body with a sharp extremity, the subsonic gas jets forming after compression in the shock wave will easily flow around the sharp edge, and the stationary shock wave will be closer to the sharp edge than in the case of flow around a blunt profile. In the case of a sufficiently small angle of aperture of the sharp edge we may expect the occurrence of the phenomenon shown in Fig. 44a, in which the shock wave touches the sharp edge. If this is the case, then by changing the scale (for example, if we switch to a projectile, the linear dimensions of which are greater by a specific factor than the bullet shown in Fig. 44a) we hardly change, if at all, the conditions at the very vertex. If we take the boundary case of an infinitely large body, we see that to find the motion near its vertex we have not, in this case, either a characteristic length or a characteristic time, and the entire motion may depend only upon the angle between the radius vector plotted at a given point from the vertex of the cone and the axis of the cone. We seek a solution in which all the quantities depend on this angle alone, i.e., are constant along each cone surface having a common axis and a common vertex, the cone being flowed around belonging to this very family of cones.

The stationary shock wave near the vertex also acquires the form of one of these cones the vertex of which coincides with the vertex of the body, and the angle of aperture depends on the angle of aperture of the conical vertex of the body. In which case can this result, which refers initially to the neighborhood of the edge of an infinitely large cone, be applied to a real projectile in which the conical head is connected (in the simplified case shown in Fig. 44a) with the cylindrical portion and the bottom of the projectile?

If the cone flowed around is sufficiently tapered and the flow moves at a sufficiently high velocity, one may expect that also after compression in the shock wave the gas velocity with respect to the surface of the cone will exceed sonic velocity. In this case, if gas velocity in the region GFABCD (Fig. 44a) exceeds sonic velocity, the change in the nature of the motion that occurs at points D, C and further (due to the fact that at these points the surface of the projectile noticeably differs from the continuation of the conical surface AB) will not affect the motion near AB, and will not move against the direction of flow. Thus, one can apply the partial solution for an infinitely large cone that depends only on one angle and is not too difficult to be computed, to plotting the motion on the entire conical section near the vertex of the projectile, on condition that this vertex be sufficiently tapered so that velocity after compression in the shock wave still exceeds sonic velocity.

On the shock wave surface we have a refraction of the flow lines. In the case of a so-called strong discontinuity, i.e., in a shock wave, only the normal velocity component undergoes a sudden change, while the velocity components tangential to the shock wave surface remain unchanged. From this follows the refraction of the flow line in the shock wave shown in Figs. 44a, b. The essential angle of the cone formed by the shock wave surface is calculated from the condition according to which after refraction in the shock wave and the subsequent bending, according to the equations of motion, the flow lines near the surface of the projectile must be parallel to the generatrices of the cone flowed around.

We shall not dwell on the details of the design, and we refer to motion and to the design diagram not so much because of the numerical results, which are far from the

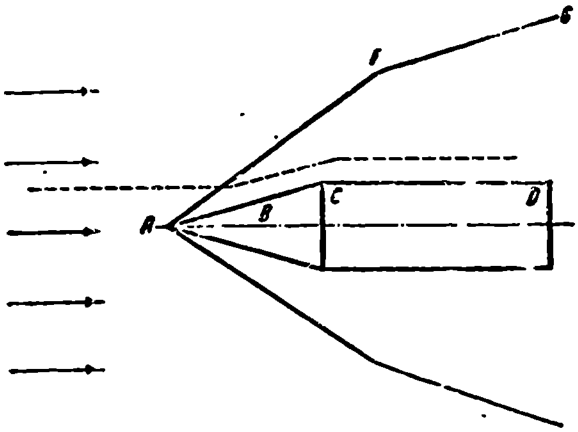

Fig. 44a.

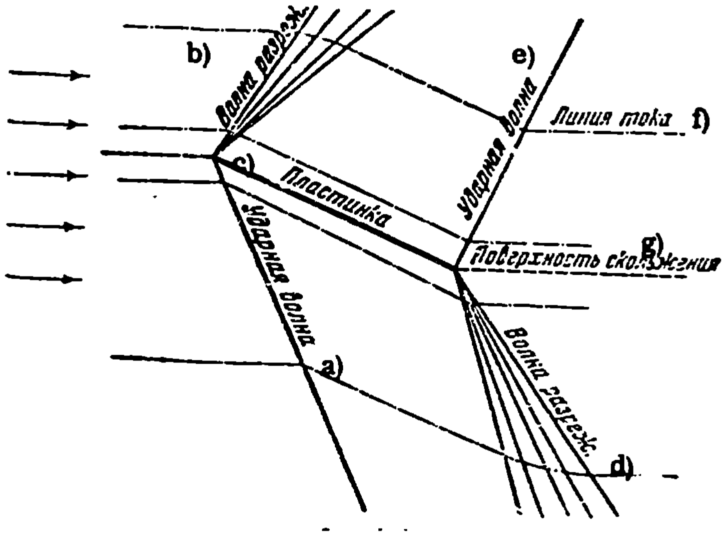

Fig. 44b.

CODE: a) Shock wave; b) expansion wave; c) plate; d) expansion wave; 3) shock wave; f) flow line; g) sliding surface.

area of application interesting us, but rather as an example of those mathematical simplifications which are specific precisely for supersonic flow and are closely related with the application of similarity theory [42]. At the present time Frankl' has developed methods for computing the distribution of pressure on the surface of pointed bodies of revolution also in those cases where they differ from a cone [26, 27].

Another, even simpler case is that of the supersonic flow about a thin plate slightly inclined in the direction of the flow (Fig. 44b).

At the front edge two waves are formed, a shock wave below the plate, in which the flow lines are suddenly refracted and following the wave move parallel to the plate, and an expansion wave above the plate, in which there gradually occurs the same bending of the flow lines.

Near the front edge the state also depends only on the ratio y/x (if the origin of the coordinates is placed at that point), as in the problem on the motion of a piston at a constant velocity the motion only depended on x/t.

The phenomena at the rear edge are similar to the propagation of an arbitrary discontinuity, since at that point two flows join, the pressures of which are different. Behind the rear edge there arise a shock wave, an expansion wave and a discontinuity of special kind (dashed line) on which now there occurs the discontinuity also of the tangential velocity component (eddy surface) . However, with sufficient flow velocity and a slight inclination of the plate the flow along the plate continues to move at supersonic velocity, and the phenomena at the rear edge have not an adverse effect on the properties of the flow near the surface of the plate. Pressure on the upper surface of the plate is less, while pressure on the bottom surface is greater than pressure in an unperturbed flow. This results in the appearance of a force that acts in a normal direction to the the plate surface in the direction upward and back. To calculate drag and lift it suffices to calculate the waves which touch the front edge.

It is characteristic that in gas dynamics of supersonic flow d'Alambert's paradox (the absence of resistance in a nonturbulent flow around a body by an ideal fluid) does not take place. There arises what we term as wave resistance, associated with the presence of steady waves which carry away the work performed by a moving body against resistance forces.

At the same time, at high velocities the irreversible heating of the substance subjected to shock compression becomes quite significant, and it remains in the form of a "trace" after the passage of the body.

The flow around a wing is thus designed from the solution of the problem of the flow around an angle formed by the wing and the flow line hitting the front edge. The flow around an angle was studied by Prandtl [77] and Meyer [71]. Graphic methods for the solution of equations that determine the parameters of oblique shock waves can be found in the general manuals [27, 23, 35, 39].

By compressing a gas that flows around a body which moves at supersonic velocity one can achieve a rapid heating of the gas to extremely high temperatures. Leypunskiy and this author tested an aluminum bullet flying at a velocity of 3,300 m/sec which crossed an area of mercury vapor where it provoked an increase in temperature up to several tens of thousands of degrees (computation, assuming constant thermal capacity, yields 45,000 degrees). An extremely strong thermal luminescence of mercury vapor on the bullet's path was also observed [125].

By shooting bullets through gases and gas mixtures subject to chemical reactions we can study the velocity of reaction at a temperature up to 4,000° and a reaction time of approximately 10^{-5} sec [104].

Chapter 18

Theory of Jet Propulsion

Modern military technology is interested in jet-propelled missiles. By complicating the design of the missile and reducing the efficiency of gunpowder as compared with conventional artillery systems one attains the substitution of the heavy gunbarrel by a light guiding rod. One also eliminates recoil. According to a course by Serebryakov [112], published before World War II, these properties of jet-propelled missiles may turn out to be useful for military operations in the mountains or for landing operations. They may also be useful for installing missiles on airplanes, motor cars, small ships, etc.

Fig. 45.

CODE: a) Gunpowder.

The diagram of a jet-propelled missile (Fig. 45) is taken from M Rua [111]. The gunpowder is contained in a chamber, and the combustion products escape under high pressure (Rua gives calculations for pressures up to 500 atmospheres) from a Laval nozzle. Computation of jet propulsion under these conditions is based on the gas dynamic theory of outflow (Chapter 3). However, in order to better acquaint ourselves with the problem and the particular features of supersonic outflow, we being with studying the simpler case of an incompressible fluid.

Let us imagine an apparatus (Fig. 46) consisting of a chamber with a simple, tapering nozzle. Pressure in the chamber is denoted by p, pressure in the ambient medium (atmosphere) is denoted by p_1, and the area of the nozzle outlet is denoted by F.

Both theory and experiments show that in a short nozzle with a smooth outline, outflow velocity satisfies very precisely Bernoulli's law and the jet fills the entire cross section. Thus

$$\frac{\varrho u^2}{2} + p_1 = p; \quad G = F\varrho u, \qquad (XVIII-1)$$

where G is the weight rate of the fluid.

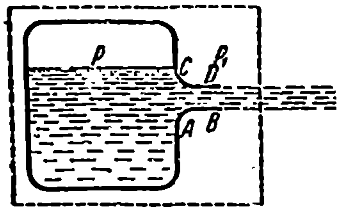

Fig. 46.

Pressure in the outlet cross section of the jet does not differ from p_1. We surround the apparatus with a control surface. The momentum acquired by the fluid during a time t is equal to the product of the outflowed amount of fluid times velocity. According to Newton's second law, the acquired momentum is equal to the momentum acting on the fluid. According to Newton's third law, the force acting from the side of the apparatus on the fluid is identical with the reaction force R experienced by the apparatus.

We assume that the direction of the force towards the left is positive (Fig. 46), and the direction of velocity to the right is positive, and obtain the equation for the momentum I

$$I = Rt = Gtu = F\varrho utu, \tag{XVIII-2}$$

$$R = F\varrho u^2. \tag{XVIII-3}$$

We substitute the velocity expression derived from Bernoulli's law and find

$$R = 2F(p - p_1). \tag{XVIII-4}$$

The result is remarkable in that from this formula there have been eliminated the quantities characterizing the properties of the fluid. The jet power is proportional to the difference in the pressure that causes outflow.

Now we approach the computation of R from another angle, and determine the resultant of pressure forces on the inner and outer surfaces of the apparatus. Let us assume that the nozzle is closed by a plug . Pressure p is acting on the inner surface of the apparatus and on the surface of the plug, while pressure p_1 is acting on the outer surface. The resultant force for a sealed apparatus (i.e., the apparatus, the nozzle, and the plug taken as a whole) is equal to zero. The force acting on plug R_3 is $R_3 = -F(p - p_1)$. It is obvious that the resultant force acting on the entire surface of the apparatus, but without the plug, is $R_1 = F(p - p_1)$ since $R_3 + R_1 = 0$. However, the expression for R given above is twice as large. This paradox is due to the fact that by removing the plug from the nozzle, force R that acts on the apparatus increases twofold as compared with force R_1 at the instant when the plug has already been removed from the apparatus but is still inside the nozzle. As the plug is removed the fluid begins to flow out. The fluid acquires momentum gradually in the tapering nozzle. According to Bernoulli's law, the motion of the fluid is accompanied by a drop in pressure. The drop in the pressure on the surface portions abutting with the opening (AB, CD) gives resultant R_2, which is equal to R_1, so that

$$R = R_1 + R_2 = 2R_1$$

We shall not go here into determining R_2. The result $R_2 = R_1$, $R = 2R_1$ holds for any smooth nozzle profile that ensures a rate coefficient equal to 1.

In evaluating the quality of the performance of the jet-propelled apparatus, it would be pointless to use the energy efficiency, i.e., the ratio of the work performed by jet power to the thermal energy of the burnt fuel or gunpowder. As a matter of fact, jet power depends on the design of the apparatus and the nozzle, and on the regime of the processes that take place in the apparatus, whereas the work performed by that force depends on the velocity of the apparatus as a whole. Hence the energy efficiency also depends on the velocity of the apparatus. With an assigned constant degree of perfection of all the internal processes, efficiency will change with the change in the velocity of the apparatus so that energy efficiency in this case is not a standard for determining the perfection of the apparatus.

An extremely important index for the quality of performance of the jet-powered device is momentum I_1, known as unit momentum, i.e., the jet momentum developed by the outflow of a unit of mass. Unit momentum is equal to the ratio of force to rate

$$I_1 = \frac{I}{M} = \frac{Rt}{Gt} = \frac{R}{G}. \qquad \text{(XVIII-5)}$$

From the above formulas we get for an incompressible fluid

$$I_1 = u = \sqrt{2(p - p_1)/\rho}. \qquad \text{(XVIII-6)}$$

Unit momentum is equal to outflow velocity when measuring all the quantities by the absolute (physical) CGS system. In an actual system the dimension of I_1 is kg of force × sec/kg of mass or, numerically, $I = u/g$, where g is gravity acceleration

For an incompressible fluid, outflow velocity and unit momentum are proportional to the square root of the difference in pressures in the chamber and in the surrounding medium. To achieve optimum effect it is desirable to increase outflow velocity by increasing the pressure differential. In the case of outflow of gas-like gunpowder combustion products under increased pressure, we run into the effect of incompressibility, into the need of using an expanding Laval nozzle and into phenomena of critical and supersonic outflow.

A Laval nozzle is characterized by two cross sections, namely, a minimal one (critical) F_k and an outlet one $F_a > F_k$. In the following we denote $F_a/F_k = \theta$. In the critical cross section we attain critical pressure which represents a specific portion of the pressure in the chamber (about 55%). Pressure p_a, attained in the outlet cross section F_a, depends on θ. Below we investigate an ideal gas having constant thermal capacity. In this case

$$\frac{p_a}{p} = \Pi_a(\theta). \qquad \text{(XVIII-7)}$$

The outflow velocity attained at the outlet of the nozzle, according to the St.-Venant-Wenttzel formula, depends on pressure. As we did it in Chapter 3 (see Fig. 6), we refer outflow velocity to sound velocity at the initial state

$$\frac{u_a}{c_0} = \varphi = \varphi(\Pi_a) = \varphi(\theta). \qquad \text{(XVIII-8)}$$

If the nozzle is so chosen that it agrees with pressure p, which exists in the chamber, then pressure in the jet in the outlet cross section p_a does not differ from atmospheric pressure p_1.

$$p_a = p_1; \ \theta = \theta\left(\frac{p_1}{p}\right). \qquad \text{(XVIII-9)}$$

In this case, the jet as it leaves the nozzle is in a mechanical equilibrium with the surrounding medium, and the velocity of the jet as it leaves the nozzle does not change ($u_a = u_1$, for the notation of u_1 see below).

We surround the apparatus with a control surface (see Fig. 46). Pressure on the control surface is equal to atmospheric pressure everywhere, including those spots where the surface intersects with the outlet cross section of the jet since, as stipulated, $p_a = p_1$. In this case the resultant of the pressure on the control surface is equal to zero. Jet power is equal to the product of the rate times the velocity at the outlet cross section of the nozzle

$$R = Gu_{a}, \qquad \text{(XVIII-10)}$$

Unit momentum is equal to outlet velocity, exactly as in the case of outflow of an incompressible fluid. The differences from an incompressible fluid amount to the following: 1) a more complex dependence of outflow velocity on pressure, and 2) the fact that to achieve the regime under investigation, for which $p_a = p_1$, we must have a specific widening of the Laval nozzle that depends on the ratio p_1/p. In an incompressible fluid the equality $p_a = p_1$ was obtained automatically, at the outflow from any nozzle, including the simplest tapering nozzle which gives the smallest losses from friction and turbulence.

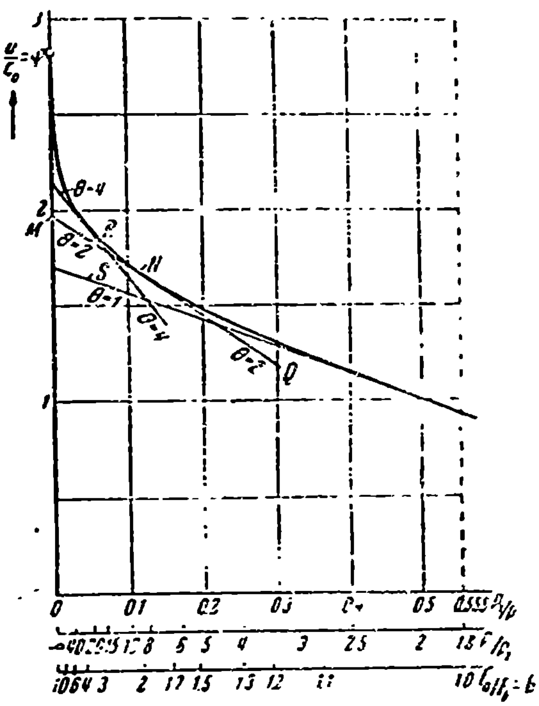

Fig. 47a.

The results from computations by the St.-Venant-Wenttzel formula for an ideal gas with an adiabatic index of 1.25 are given graphically in Figs. 47a and 47b. The value K = 1.25 was obtained by D.A. Frank-Kamenetskiy for the combustion products of smokeless gunpowder. On the ordinate (Figs. 47a and 47b) are marked the values for $\varphi = \frac{u}{c_0}$, and on the abscissa we find the values for the ratio p_1/p. The corresponding values for θ and p/p_1 are also marked on the abscissa.

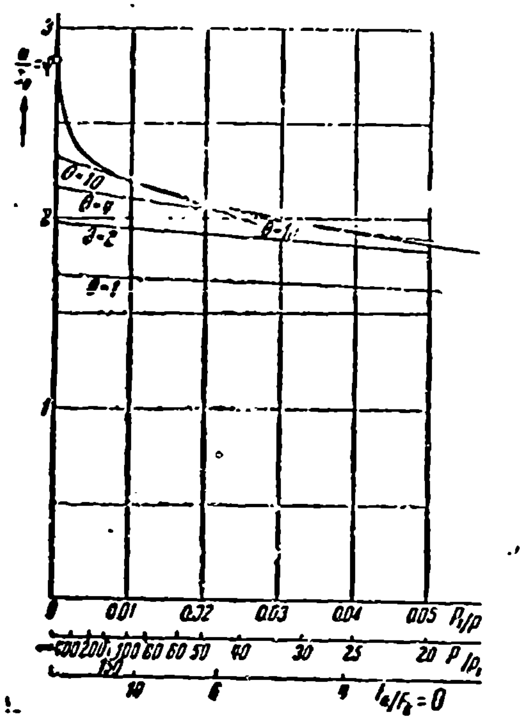

Fig. 47b.

With assigned p_1 (atmospheric pressure) and pressure p in the chamber we set up the ratio p_1/p, find on the upper scale the corresponding abscissa value, and on the bottom scale we find θ. Outflow velocity and unit momentum are read on the heavy line φ. Thus

$$I = c_1 = u_a = \varphi c_0. \qquad \text{(XVIII-11)}$$

According to internal ballistics it is customary to characterize the state of gunpowder combustion products by gunpowder power $f = \frac{p}{\varrho}$. We neglect the deviations from the laws of an ideal gas and find

$$c_0 = \sqrt{kf}, \qquad \text{(XVIII-12)}$$

so that

$$I_1 = u_1 = \varphi\sqrt{kf}. \qquad \text{(XVIII-13)}$$

We go over to technical units and write

$$I_1 - \frac{\text{kg. sec}}{\text{kg}},$$

$$f - \frac{\text{kg}}{\text{dm}^2} \Big/ \frac{\text{kg}}{\text{dm}^3}$$

and, substituting k = 1.25, g = 981 cm/sec^2, we find

$$I_1 = 0.113\,\varphi\,\sqrt{f}. \qquad \text{(XVIII-14)}$$

Thus, for smokeless gunpowder with f = 1,000,000 kg/dm^2/kg/dm^3 at a pressure in the chamber p = 100 kg/cm^2 and atmospheric p_1 = 1 kg/cm^2, we find

$$\varphi = 2.2,\ I_1 = 0.113 \cdot 2.2 \cdot 1000 = 250\,\frac{\text{kg/sec}}{\text{kg}}.^{43}$$

The value of φ is read on the diagram in Fig. 47b in which the region p_1/p most interesting from a practical point of view, ranging from 0 to 0.05 (p from 20 kg/cm^2 up), is magnified.

By substituting the expression of flow rate for critical outflow we express jet power by the critical cross section and pressure in the chamber (the subscript k refers to the quantities in the critical cross section)

$$R = G u_a = F_k \varrho_k u_k u_a = F_k \varrho_0 \frac{\varrho_k}{\varrho_0} \cdot c_0^2 \cdot \frac{u_k}{c_0} \frac{u_a}{c_0} =$$

$$= F_k p \cdot \frac{\varrho_0 c_0^2}{p} \frac{u_a}{c_0} \frac{u_k}{c_0}, \qquad \text{(XVIII-15)}$$

$$R = \text{const}\ \varphi \cdot F_k p = 0.74\,\varphi\,F_k p. \qquad \text{(XVIII-16)}$$

The numerical coefficient is found for the adiabatic exponent 1.25, for which Figs. 47a and 47b have been plotted. As in the case of an incompressible fluid, the last expression does not contain gas density, gas temperature and similar quantities. In the French literature the dimensionless ratio $R/F_k p$ is termed "coefficient de propulsion" (propulsion coefficient) (Serebryakov, Greten, Oppokov [112]).

In the example given

$$(\varphi = 2.2,\ R = 0.74 \cdot 2.2 \cdot F_t p = 1.63 F_t p)$$

this coefficient reaches 1.63. In the case of outflow of an incompressible fluid referred to the pressure differential $p - p_1$, the coefficient was equal to 2.

What is the nature of the motion and how to compute jet power in the case where the widening of the nozzle θ does not correspond to the pressure ratio? The gas jet flows out at supersonic velocity into the surrounding medium at a pressure in the jet in the outlet cross section p_a, that differs from atmospheric pressure p_1. At the point of contact, on the edge of the outlet cross section, the flow becomes perturbed. It widens, accompanied by an increase in velocity in the case of $p_a > p_1$, or it is compressed, accompanied by a decrease in velocity in the case of $p_a < p_1$. The progressive motion of the gas in the jet is added to the propagation of pertubations from the edge of the cross section to the axis of the jet. Owing to this, the surface on which individual flow lines are subject to disturbance, acquires the shape of a cone that leans on the outlet cross section and extends in the direction of the jet (see Fig. 49 below). In the outlet cross section proper, the flow is unperturbed, pressure is equal to p_a everywhere and outflow velocity is u_a everywhere. The state of the flow in the outlet cross section depends on the state of the gas in the chamber and the widening of nozzle θ, according to the formulas. The state of the flow, and, in particular, the quantities p_a and u_a, are completely independent of atmospheric pressure p_1. This is obvious from the fact that the perturbation caused by the difference between p_a and p_1 does not propagate into the outlet cross section.

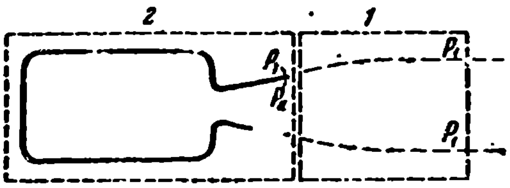

Fig. 48.

Again we surround the apparatus by a control surface which passes through the outlet cross section (surface 2, Fig. 48). Everywhere except at the outlet cross section of the nozzle F_a pressure is equal to p_1, but in F_a pressure is equal to p_a. The resultant pressure force is equal to $F_a(p_a - p_1)$. In calculating jet power we must add this quantity

$$R = Gu_a + F_a(p_a - p_1). \qquad \text{(XVIII-17)}$$

We substitute

$$G = F_a \varrho_a u_a, \qquad \text{(XVIII-18)}$$

and transform

$$R = G\left(u_a + \frac{p_a - p_1}{\varrho_a u_a}\right) = Gu_1, \qquad \text{(XVIII-19)}$$

We introduce the quantity u_1 which we define as follows:

$$I_1 = u_1 = u_a + \frac{p_a - p_1}{\varrho_a u_a}. \qquad \text{(XVIII-20)}$$

This quantity represents the mean value of axial velocity of the jet where the pressure in the jet has become equal to atmospheric pressure. This can be proved by setting up the momentum equation for the control surface 1, Fig. 48, which is entered by the jet at pressure p_a and velocity u_a, and which the jet leaves at a pressure p_1 and the velocity u_1 sought.

It follows from this equation that unit momentum for $p_a \neq p_1$ is precisely determined by velocity u_1 and not by outflow velocity u_a.

It can be shown in a general form that for a given initial state of the gas in the chamber and a given p_1, u_1 reaches a maximum when $p_a = p_1$. In other words, the most expedient case is precisely the one examined by us earlier which involves a complete widening of the nozzle until pressure reaches atmospheric pressure.

To prove this we set up the derivative of Eq. (XVIII-20)

$$\frac{du_1}{dp_a} = \frac{du_a}{dp_a} + \frac{1}{\varrho_a u_a} - \frac{p_a - p_1}{(\varrho_a u_a)^2} \frac{d(\varrho_a u_a)}{dp_a}. \qquad \text{(XVIII-21)}$$

According to Bernoulli's law (see Chapter 3), by differentiating Eq. (III-9) we find

$$\frac{du_a}{dp_a} = -\frac{1}{\varrho_a u_a}. \qquad \text{(XVIII-22)}$$

For $p_a = p_1$, $\frac{du_1}{dp_a} = 0$;   We can readily show, be determining the sign of

$$\left.\frac{d^2u_1}{dp_a^2}\right|_{p=p_1} = -(\varrho_a u_a)^{-2}\frac{d\varrho_a u_a}{dp_a} < 0,$$

that here we are precisely dealing with a maximum of u_1.

This result is perfectly natural. By examining the pressure on the conical surface of the widening portion of the Laval nozzle, we satisfy ourselves that when $p_a > p_1$ the lengthening of the cone (together with an increase of F_a and a decrease of p_a) yields an additional term that increases jet power. When $p_a < p_1$ the lengthening of the cone yields a term that reduces jet power. We remind the reader of the remark in Chapter 3. In all cases the jet, sooner or later after its outflow, acquires a pressure p_1. However, in the case of $p_a \neq p_1$ a portion of the pressure differential is expended for radial velocity components which do not create jet power.

From a practical point of view, a careful adjustment and control of the nozzle, especially in processes involving varying pressure in the chamber, for the purpose of continuously upholding $p_a = p_1$, are extremely complex. Of practical interest is the study of the performance of a jet-powered apparatus with an assigned constant nozzle, i.e., an assigned Θ with variable pressures p and p_1.

Equations (XVIII-13) and (XVIII-16), set up earlier, will keep their validity if, instead of velocity at the outflow of the jet u_a, we substitute the effective velocity u_1, given by Eq. (XVIII-20). Instead of dimensionless velocity $\varphi = u_a/c_0$ one should use $\varphi_1 = u_1/c_0$. The quantity φ_1 is a function with two variables Θ and Π_1, where $\Pi_1 = p_1/p$

$$\varphi_1 = \varphi_1(\Theta, \Pi_1). \qquad \text{(XVIII-23)}$$

Function φ_1 is closely connected with function φ. From the foregoing we can establish the following properties of φ_1:

1) If Θ is constant, function φ_1 is linearly dependent on Π_1;

2) If $\Theta = \Theta(\Pi_1)$, i.e., in the case of a widened nozzle, corresponding to the ratio of atmospheric pressure to pressure in the chamber, $\varphi_1 \equiv \varphi$ by definition.

3) If $\theta \neq \dot{\theta}(\Pi_1)$, $\varphi_1(\dot{0}, \Pi_1) \dot{<} \varphi(\Pi_1)$.

From this it follows that in the plane as shown in Figs. 47, a, b (see above) the dependence of φ_1 on Π_1 is given, for an assigned constant Θ, by a straight line that touches the curve at that value of Π_1 which corresponds to the given Θ.

Figure 47 a, b shows a number of lines φ_1 (θ = const, Π_1) for Θ = 1, 2, 4, and 10. In order to find, for example, φ_1 (2; 0.05), we look for Θ = 2 on the bottom scale of Θ, below the abscissa. The Θ - scale has been plotted in accordance with the Laval nozzle theory, so that every Θ is placed under the corresponding Π_1; $\Pi_1(\theta = 2) = 0.115$. On the curve φ we find the corresponding point N and plot the tangent MRNQ (the tangent is labelled Θ = 2).

This tangent represents the function φ_1 (2, Π_1). For Π_1 = 0.05 we find the point R, $\varphi_1(2; 0.05) = 1.84$. It is interesting to compare this value with the value of φ for an optimal widening of the nozzle for the given $\Pi_1: \theta_1(0.05) = 3.5$; $\varphi(0.05) = 1.91$. The optimal nozzle yields a gain of 3.7%. Conversely, if one takes a nozzle without diffuser, Θ = 1, one would obtain with $\Pi_1 = 0.05$, $\varphi_1(1; 0.05) = 1.63$ (point S), a quantity that would be 15% less than optimal. As we see from the foregoing, the jet momentum is proportional to the quantities of φ_1 ((Eqs. (XVIII-13), (XVIII-16)).

For the sake of convenience the diagrams give also the scales for $1/\Pi_1 = p/p_1$. This quantity represents pressure in the chamber in the case in which p_1 = 1 atmosphere absolute.

Let us now take a closer look at the outflow from the nozzle with $p_a \neq p_1$.

If $p_a > p_1$, the conical expansion wave (Fig. 49, lines a and b) at the edge of the nozzle is similar to the expansion wave at the edge of a thin plate placed into a supersonic flow (see Fig. 44b, top left or bottom right portion). The surface a, on which pressure begins to drop, propagates at sonic velocity c_a along the gas that moves at a velocity u_a. Hence the generatrix of cone a forms with the flow direction the Mach angle, $\sin \alpha = \frac{c_a}{u_a}$. Sound velocity and the direction of flow after expansion are such that the subsequent characteristics form a more elongated external cone (b, Fig. 49). Pressure drop and change in velocity occur in the layer between surfaces a and b.

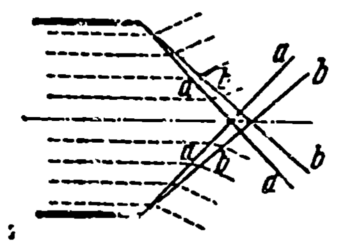

Fig. 49.

If $p_a < p_1$, the gas flowing out from the nozzle is subjected to compression by a shock wave which also has the shape of a cone. Since the velocity of the shock wave is greater than sound velocity and depends on its amplitude, the Mach angle of the wave is the greater and the cone the lower, the higher is pressure p_1. Finally, for some p_1 the wave velocity is comparable to outflow velocity $D = u_a$. In the outlet cross section of the nozzle a plane shock wave is formed. At an even higher pressure p_1 at the outlet, the shock wave "hides" inside the diffuser of the Lavel nozzle. In the shock wave, the supersonic flow changes into a subsonic flow. Pressure in a subsonic flow in the wide part of the nozzle increases as the gas moves, since velocity decreases and, in the terminology of hydraulic engineers, the kinetic head changes into pressure. Beginning with that value of p_1 at which the shock wave moves inside the nozzle and changes the distribution of pressure on the surface of the nozzle, the equations and nomograms derived above for determining jet power are no longer valid.[44]

Figure 50 shows experimental pressure distribution curves on the axis of a Laval nozzle through which water vapor is blown at varying counter-pressure at the nozzle outlet.

The curves have been taken from the turbine designer Stodoli, who also investigated and treated the abrupt increase in pressure as a Riemann-Hugoniot-Rankine shock wave.

By combining the laws of adiabatic flow (Chapter 3) with the concept of a shock wave inside or at the outlet of a nozzle, it became possible for us to determine the outflow regime for any pressure at the nozzle outlet between p_4 and p_5 (Fig. 11, Chapter 3).

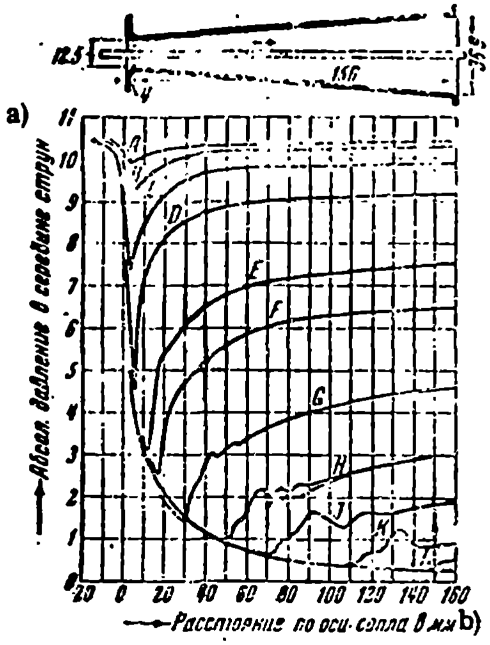

Fig. 50.

CODE: a) Absolute pressure in the center of the jet; b) distance along the nozzle axis, in mm.

Chapter 19

Reflection of a Shock Wave

Let us imagine a solid placed in a space in which a shock wave propagates. At the instant when the wave front reaches the solid, motion changes in comparison with the motion involved in the propagation of a shock wave in free space. Let us clarify the peculiar features of that motion, which determine the forces acting upon the solid.

Belyayev [2] at the Institute of Chemical Physics studied experimentally the conditions that arise when shock waves are reflected or collide. He evaluated the pressure increase from the reflected of a wave by comparing the buckling of two lead membranes, one of which was placed tangentially and the other normally to the direction of the wave caused in the air by the detonation of a TNT charge. In Fig. 51a the membrane disrupts only slightly the conditions of propagation of a shock wave, and the magnitude of pressure p can be measured by its buckling. Conversely, it is obvious that the force acting on the membrane placed normally to the wave direction (Fig. 51b) depends also on the velocity of the gases in the shock wave. Becker [38] following Rüdenberg [83], kept this fact in mind and introduced the sum $F = p + \varphi u^2$ as the characteristic of the wave momentum.

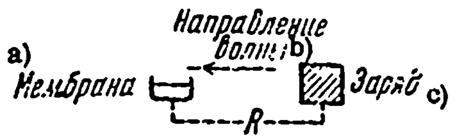

Fig. 51a.

CODE: a) Membrane; b) direction of wave; c) charge.

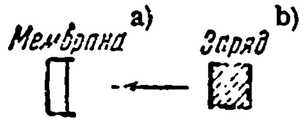

Fig. 51b.

CODE: a) Membrane; b) charge.

Rüdenberg takes pressure to be 2F in the case of a normal impact against an obstacle. However, the introduction of 2F is, strictly speaking, not justified. Vlasov [3] correctly notes that this quantity differs by 50% from the true value of pressure.

Let us investigate the conditions at the instant when in the test shown in Fig. 51b the shock wave reaches the membrane. By changing the reading system, we may say that at this instant the membrane begins to move at a velocity u with respect to the gas compressed in the shock wave. This motion of the membrane produces a second shock wave which propagates toward the first shock wave in the gas compressed by the first wave.

The first effect of the shock wave on the surface of the obstacle, which is perpendicular to the direction of the wave, is determined precisely by pressure p_1 in the counter-wave which stops gas motion near the obstacle.

Izmaylov (we quote from Belyayev's paper [2][45] whence we have also taken Figs. 51-53) devised a general formula for pressure p_1 at an arbitrary amplitude of pressure p in a incident (first) shock wave and an initial atmospheric pressure p_0

$$p_1 = p\frac{(3k-1)p-(k-1)p_0}{(k-1)p+(k+1)p_0} \qquad \text{(XIX-1)}$$

and for k = 1.4

$$p_1 = p\frac{8p-p_0}{p+6p_0}. \qquad \text{(XIX-2)}$$

In the case of a small amplitude we get an acoustic result

$$p_1 - p_0 = 2(p-p_0). \qquad \text{(XIX-3)}$$

In the case of a very large amplitude , $p \gg p_0$ we reach the limit value

$$p_1 = \frac{3k-1}{k-1}p; \text{ with } k=1.4, \quad p_1 = 8p. \qquad \text{(XIX-4)}$$

Belyayev points out that the conditions in a case of collision of two identical shock waves (see Fig. 52) do not differ from those under which a shock wave is reflected by a wall.

Within the precision limits of the test, Beleyayev's experiments corroborated Eq. (XIX-2) for both reflection and collision. The results of the tests are compared to Eq. (XIX-2) in Fig. 53.

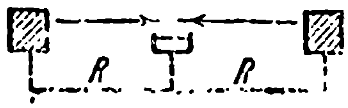

Fig. 52. Measurement of pressure upon collision of two shock waves.

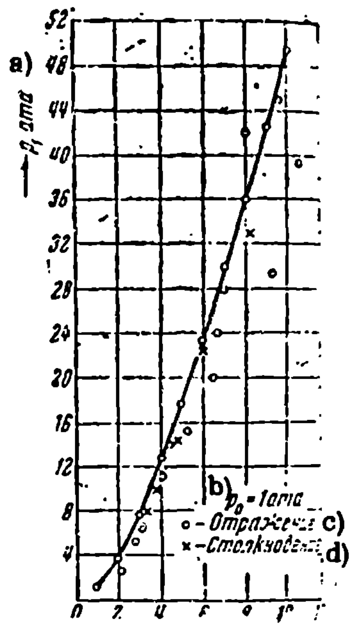

Fig. 53. Dependence of pressure during reflection and pairwise collison of shock waves on the amplitude of the shock wave (Measurements by A. F. Belyayev).

CODE: a) p_1 atmospheres absolute; b) p_0 = 1 atmosphere absolute; c) reflection; d) collision.

In the case of reflection of the shock wave by the membrane, during the first instant there appears a reflected wave that moves away from the membrane. In the absence of lateral walls this moving away of the wave must lead to its weakening, and during a time of the order of d/c, where d is the diameter of the membrane, we must get a transition to a steady flow around the obstacle at a velocity u. We have to point out a very significant fact, namely, that the velocity of the gas compressed by a powerful shock wave exceeds sound velocity in the compressed gas. Thus, in the case of a steady air flow around a body caused by a powerful shock wave we will obtain a transition similar to the one described earlier in Chapter 17, with a stationary shock wave in front of the obstacle (Fig. 54). However, the amplitude of the stationary shock wave is less than the initial value of the amplitude of the reflected wave, since in the stationary wave $D_1 = u$, whereas in the reflected wave $u_1 = u$. Steady pressure on the membrane surface in the limit case of an extremely powerful wave in a diatomic gas is

$$p_2 - 5.24\ p, \qquad \text{(XIX-5)}$$

instead of the initial value equal to $p_1 = 8_p$ of Eq. (XIX-4).

If the incident shock wave is weak, then, as before, at the instant of incidence there is formed a reflected wave. In the case of small amplitude, Eq. (XIX-3) yields

$$p_1 = p_0 + 2(p - p_0) = p + \rho u c, \qquad \text{(XIX-6)}$$

but after that the reflected wave rapidly weakens and fades into infinity. Steady pressure is computed by Bernoulli's formula

$$p_2 = p + \frac{\rho u^2}{2}. \qquad \text{(XIX-7)}$$

Computations show that if k = 1.4, to attain sonic velocity in a shock wave p must equal 4.5 p_0.

With $p/p_0 < 4.5$, $u < c$ a spherical wave is formed (Fig. 55) which separates from the obstacle. The amplitude of the shock wave can be determined by means of flash photography (Fig. 55). We shall not dwell here on the details of the computation.

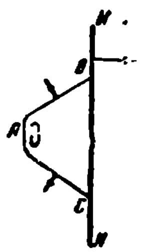

Fig. 54. Front of an acoustic wave ABC, that arises in a compressed gas during the passage of an extremely powerful shock wave MN past a small obstacle. In the shock wave MN supersonic velocity of the compressed substance is achieved. Segment AB is the cross section of a Mach cone (see Fig. 12b).

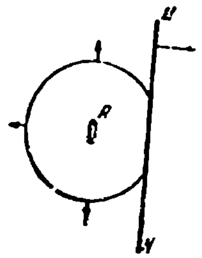

Fig. 55. Spherical front of an acoustic wave generated in a compressed gas by a weak shock wave MN past an obstacle A. The amplitude of wave MN is not sufficient to attain supersonic velocity (see Fig. 12).

In the pressure interval in the wave from 5 p_0 to 10 - 15 p_0 the measurement of the dip angle of Mach waves on a flash photograph (see Fig. 54) may serve for a precise determination of the instant parameters of an incident shock wave.

Let us note, finally, that supersonic velocity of a compressed gas does by no means contradict the general theory which requires that $D < c + u$. In powerful shock waves, beginning with $p/p_0 = 4.5$ upward, perturbation is not transmitted against the direction of flow of the gas, but any perturbation in the back is transmitted to the wave front.

Duhem [48] made it a point to note that in a shock wave in which density increases more than $2/(k - 1)$ times (which corresponds to a pressure increase $p > \frac{4k - k^2 + 1}{(k - 1)^2} p_0$, i.e., $p > 15.25\,p_0$ for $k = 1.4$), the propagation velocity of the shock wave with respect to the unperturbed gas is greater than sound velocity in the compressed gas, $D > c$. However, so far as we know, during a passage through $D = c$, there arise no special features in the behavior of the wave.

Chapter 20

The Effect of Explosives. Introduction

One of the most important areas of application for the theory of shock waves are explosives, explosions and their effects.

An explosion is a quick chemical reaction during which the explosive is entirely or partially converted into a gas of more or less high temperature.

Depending on the composition and the state of the substance, on the conditions under which the explosion takes place and on the conditions that cause the explosion, the chemical reaction takes place in different ways at varying velocities.

Only an extremely fast chemical reaction leads to extremely wide differences in pressure and propagation of shock waves, which represents a particular feature of the explosion.[46] For this reason we are particularly interested in the problem concerning the speed of the chemical reaction.

Any practically applicable explosive is chemically inert at room temperature. The chemical reaction, the explosion, occurs only after ignition (priming) of the explosive.

As a rule, the explosive is ignited at one spot only. The complex processes under investigation result in the fact that the chemical reaction in one layer provokes a chemical reaction in the neighboring layer, and so on. As a result we have the propagation of the chemical reaction at a specific linear velocity (the dimension of that velocity is length x $time^{-1}$) in the space covered by the explosive.[47]

There arise two problems : one concerning the conditions and propagation rate of the reaction, and the other the distribution of pressure and other quantities in space at the instant the reaction is completed. The theoretical investigation of these problems exceeds the scope of the present monograph. Experience shows that in high explosives produced by modern technology, the propagation rate of the reaction reaches several thousands of meters per second and exceeds sound velocity in these substances. For this reason, in the case of central ignition, the outer portions of a high-explosive charge have no time to move from their places until the explosion is over. Mean density of explosion gases is equal to initial density of high explosives. Mean temperature of explosion products ranges from 1500 to 4000°K, depending on the type of high explosive.

According to Clapeyron's law, mean density 1.3, mean temperature 3000°K and mean molecular weight of the explosion product 25 should correspond to a pressure

$$\bar{p} = \frac{1.3 \cdot 3000 \cdot 22\,400}{273 \cdot 25} = 13\,000 \text{ atm}$$

In actual fact (because the gas is not ideal), however, mean pressure is several times higher. Moreover, reaction propagation results in an irregular distribution of pressure in the volume taken up by the explosion product. A part of the explosion product is in motion. The irregularity and the motion of the explosion product can be understood if one considers that different particles of the explosive react at different times. Taking this into consideration, maximum pressure in an explosion product attains 100,000 to 400,000 atmospheres.

As the reaction is compiected, the explosion products, the state which is described above, are surrounded by an unperturbed atmosphere. The expansion of the explosion products is accompanied by the formation of a powerful shock wave.

During expansion, the explosion products cool off close to room temperature. They cover a volume which, as an average, exceeds 1000 times the volume of the explosive.

Objects placed at a distance up to 10 radii of the charge are subject not only to the effect of the shock wave propagating in the air, but also to the effect of the expanding explosion products.[48]

Near the charge, while expansion is negligible and temperature and density of the explosion products are therefore great, a considerable thermal effect on the surface of the obstacle is quite characteristic.

Frequently explosion products contain carbon monoxide and hydrogen, especially in the case of explosives with a negative oxygen balance: the combination of carbon monoxide and hydrogen of the explosion products with the oxygen from the air is not only possible, but probable. In the case of TNT, combustion temperature of the explosion products (carbon monoxide and hydrogen of the explosion products) in air oxygen attains 220% of explosion heat (the heat generated by the conversion of the explosive into explosion products).[49]

At the present time it is not understood how and when there occurs a reaction of CO and H_2 contained in the explosion products with air oxygen, and to which extent the energy from the reaction is used as the mechanical energy of the explosion.[50]

As they expand, the explosion products act as a piston and push the air in front of them. A good (close to 1) efficiency in utilization of chemical explosion energy during the first stage of the process, corresponds to the considerable expansion of explosion products.

The propagation of the shock wave due to the irreversible nature of compression in the wave is accompanied by the dissipation of mechanical energy and its conversion into thermal energy. For this reason, it also accounts for the fact that as the wave propagates its surface and the amount of substance involved in the motion increase, and the wave's amplitude drops with distance.

Finally the wave reaches the obstacle. On the one hand, the wave is reflected and moves around the obstacle. This is a phenomenon that occurs in air and determines the force acting upon the obstacle. On the other hand, this causes the displacement and deformation of the obstacle, i.e., it causes those processes which, in the final analysis, determine the toppling or destruction of the obstacle.

We are facing here two typical cases. In the first case the action is determined by peak pressure; if peak pressure is not sufficient to destroy the obstacle, the subsequent effect of weaker pressure will not change anything. This occurs when maximum force and deformation are attained very rapidly in the system to be destroyed, during a time less than the time during which pressure drops. An an example we can take the destruction of a solid steel plate by a charge placed on its surface.

Destruction depends on maximum pressure, i.e., on the type of explosive and the distance (gap) between the charge and the surface.[51]

In the second case (which occurs more frequently), the shock wave action time is short as compared with destruction time. For example, we take the toppling of a brick wall 1.5 meters high and 0.25 m wide (Fig. 56). To achieve this one must impart a velocity of about 0.5 m/sec to the wall's gravity center. At such a velocity it will take about 0.25 sec for the gravity center to reach the highest point (which corresponds to the position of the wall shown by the dashed line).

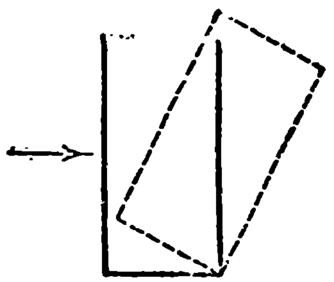

Fig. 56.

It is obvious that the action time of the shock wave is considerably less than 0.25 sec. In fact the wave covers about 100 m during 0.25 sec. Consequently, during the shock wave action time the displacement of the wall is negligible, the wall only gathers velocity and with that velocity motion continues by inertia until the final action of the wave. The acquired velocity does not depend on the magnitude of peak pressure, but on the area of the pressure-time curve, i.e., on the pressure momentum, which determines whether or not the wall be toppled.

If an elastic structure, e.g., one consisting of long metallic rods, is to be destroyed, then, compared with destruction time (the time of deformation required for destruction), the action time of the wave will also be small as will the shifts and displacements occurring during that time. The maximum deformations dangerous for the structure arise later, after a time equal to one-fourth of the period of the system's proper oscillation. Shock wave pressure at that time no longer acts on the system, and deformation occurs by inertia on account of the velocity gathered from the beginning.

Later on, when investigating the propagation of shock waves from the detonation of explosives, we will have to study the change of both parameters that characterize a shock wave, namely, maximum pressure and general momentum. The ratio of the momentum to maximum pressure characterizes the actual action time of shock wave pressure.

Of great importance is the interaction between wave and object when measuring pressure and momentum, or the effect of a shock wave on an object. We have seen above (Chapter 19), that due to reflection, pressure on a surface placed normally to the wave front exceeds several times that exerted on a surface placed tangentially to it. Furthermore, the force momentum depends on how the air compressed in the wave flows around the object. Hence the relationship between the pressure momentum of the wave[52] and the force momentum experienced by the object also depends on the ratio of the action time of shock wave pressure to the time the wave flows around the object.

Chapter 21

Simulation of an Explosion and of the Propagation of Blast Waves

The complexity of analytic computation of even the simplest symmetric and schematic problems requires the establishment of a method of simulating explosions and their effect on a small scale, and the determination of laws governing the application on a large scale of the results obtained on a small scale. In other words, it becomes necessary to establish laws of similarity.

In Chapters 6 and 16 we have seen that gas dynamics equations contain only a specific characteristic velocity (sound velocity) but do not contain either length [distance] or time. In Chapter 16 we showed that in the propagation of shock waves the introduction of dissipative quantities does not introduce a characteristic length. Hence there is the possibility of setting an arbitrary scale in the case of simulation. Similarity will be ensured if all the dimensions are changed in accordance with the rules of geometric similarity.

If we investigate the problem of the propagation of explosion pressure, for reasons of similarity, it is also necessary that the properties of the explosion products be in a certain relationship with the corresponding quantities characterizing the properties of air. This refers to sound velocity, the density the pressure of explosion products and of the air.

Since the properties of air under atmospheric pressure are known and constant, similarity will be maintained if we maintain the properties of the explosion products.

In order to maintain the properties of the explosion products it is necessary to fulfill two conditions, the first of which is the conservation of the properties of the explosive.

This is a very simple condition. During simulation one must use the same explosive with the same charge density as if it were the case of an actual explosion. This condition is necessary but not sufficient. It is also necessary that this similarity not be violated during the explosion process, i.e., during the process of the chemical reaction.

It cannot be expected that the similarity will be maintained in full. A chemical reaction is characterized by a specific rate, i.e., by a specific time required for its completion. However, it has been mentioned many times that gas dynamic phenomena have a charactersitic propagation velocity in cm/sec. For this reason, as we change the geometric scale of the test, all the times change proportionally. For instance, if a model is one-tenth the size of the actual charge, then the time for the passage of the shock wave from the charge to the obstacle is one-tenth that of the actual one. As we change the scale, there occurs a change in the ratio between the reaction time and other times which depend on the motion of the gas. This, generally speaking, violates similarity.

It has been known for a long time that blast velocity measured for explosives of small-diameter shells, turns out to be reduced with respect to normal values measured in large-diameter shells. Blast velocity depends on the size (as this is required by similarity), only beginning with a specific, sufficiently large diameter.

A striking expression of the violation of similarity is encountered in an investigation performed by Yu.B. Khariton et al. [116], who studied the phenomenon of a critical diameter (for the blast): charges of liquid nitroglycerin enclosed in pipes of a large diameter detonate (with due priming), but in very narrow pipes detonation "dies off" and therefore does not propagate.

It is obvious that as we measure shock waves of charges with varying diameters, even if all the other conditions (of geometric similarity) are kept, we will get completely different results if the critical diameter of the pipe is exceeded.

As the blast propagates, new layers of the explosive are involved by the layers transformed earlier into explosion products by the chemical reaction. At the present time, the part played by various factors (the effect of high pressure on the reaction rate, heating from compression, heating of the explosive from contact and mixing with explosion products, etc.) is not quite understood. All we know is that the layer involved in the reaction is subjected to the effects of high pressure. Explosives contained in a fragile glass tube or any other thin shell tend to fly apart in all directions under high pressure.

According to Yu. B. Khariton, the damping of the blast of an explosive having a small diameter is due precisely to the fact that the scattering time of the charge becomes less than the chemical reaction time. The explosive is scattered, and pressure drops before the actual reaction has a chance to take place. With a larger diameter, scattering time of the charge increases, too. If it exceeds the chemical reaction time, a nondamped blast becomes possible.

The existence of a critical diameter violates the similarity of the explosion of charges of various sizes. On the other hand, it gives us a criterion by which to determine the conditions in the region in which the similarity is to occur.

To obtain similarity it is necessary that the reaction rate be low as compared with other characteristic times. From Yu. B. Khariton's critical diameter theory we can conclude that the explosion of two charges of the same shape but different size will be similar to one another if all the dimensions of the smaller charge (and, therefore, the dimensions of the larger charge) exceed several times the critical diameter.

It must be attempted to obtain a complete blast both in the larger and smaller charges. Similarity is violated where completeness of the reaction increases with an increasing charge. But the completeness of a reaction cannot exceed 100%, hence we can assume that starting with a certain sufficiently large dimension similarity will be maintained.

Suppose that similarity is maintained. How do we project the data gathered from a model on events taking place in nature? All geometric dimensions are reduced to scale. We select as a characteristic dimension a charge with a radius R. Similarity points, i.e., those where all phenomena evolve in a similar fashion, will be points the distance of which, from the center of the charge, are in the same relation to the radii of the charges, that is, points at which are equal the ratios x/R, y/R, x/R or (in the case of spherical symmetry) r/R, where r is the distance of the point from the center of symmetry.

Pressure in similar systems is identical since atmospheric pressure of the air is identical, and maximum pressure of the explosion products is identical, which follows from the identity of explosion product density and explosion temperature. As already mentioned, the time in similar systems is proprotional to their dimensions. Hence, if we compare the curves of the dependence of pressure on time, we will find that they are transformed

$$\frac{t}{R} = \text{idem}, \quad p = f\left(\frac{t}{R}, \frac{r}{R}\right). \tag{XXI-1}$$

In order to deal with a dimensionless function, we write the above formula as follows

$$p = p_0 \cdot f\left(\frac{tc_0}{R}, \frac{r}{R}\right), \tag{XXI-2}$$

where p_0 is characteristic pressure (for instance, atmospheric pressure), c_0 is characteristic velocity (for instance, sound velocity in the air).

We are interested above all in two quantities, maximum pressure and total pressure momentum. We find for these quantities

$$p_{\max} = p_0 \pi\left(\frac{r}{R}\right);$$

$$i = \int (p - p_0)\, dt = \int p_0 \left[f\left(\frac{tc_0}{R}, \frac{r}{R}\right) - 1\right] \cdot \frac{R}{c_0}\, d\left(\frac{tc_0}{R}\right) = \tag{XXI-3}$$

$$= p_0 \frac{R}{c_0} \cdot \eta\left(\frac{r}{R}\right).$$

Maximum pressure at similar points is identical, and pressure momentum at similar points is proportional to the scale of R. Completely analogous formulas also hold for the motion of gases. At similar points maximum gas velocity is identical, the curves of velocity change with time are similar, and the displacement of particles is proprotional to the scale of R.

Pressure on the surface of the obstacle differs from pressure in the shock wave and depends on reflection and the flow around the obstacle by the wave. If the obstacles are similar, these phenomena will also be similar. Maximum pressure on the surface of the obstacle differs from maximum pressure in the shock wave by a factor that depends on the amplitude of the wave (see Chapter 19), that is, on p/p_0. Thus, maximum pressure of a reflected wave depends only on the ratio of the lengths of r/R. A formula like the one for pressure momentum refers in exactly the same way to the momentum acting on a unit of surface of the obstacle, so that the momentum per unit area on similar surfaces is proportional to the dimension of the charge.

It is the task of experimental and theoretical investigation to determine pressure as a function of two variables of f (tc_0/R, r/R). This is an extremely complex problem, hence it is expedient to determine first of all two functions of dimensionless distance π, η, which characterize maximum pressure and total momentum. We try to determine them in a freely propagating wave, and we also seek these functions in the presence of a specific, standard type of reflection and flowing around. Thus, Sadovskiy used instruments built into a high, solid wall. In this case, obviously, we are dealing with the reflection of a shock wave without flow around the obstacle.

For a substance with a specific density, the radius simply depends on the weight of the charge. The investigators give their data in the form of a dependence of pressure and momentum on the distance r and the mass of the charge m. Since m ~ R^3, similarity theory leads to the following dependences

$$p_{max} = f\left(\frac{r}{\sqrt[3]{m}}\right); \quad i = \sqrt[3]{m} \cdot h\left(\frac{r}{\sqrt[3]{m}}\right). \qquad \text{(XXI-4)}$$ [53]

Finally, in a moderately wide pressure change interval, it is natural to seek the definition of the quantities to be determined as power functions of the weight of the charge and the distance

$$p_{max} = \text{const} \cdot r^a m^b; \quad i = \text{const} \cdot r^c m^d.$$

The similarity laws connect the exponents. From Eq. (XXI-4) it follows that

$$a + 3b = 0; \quad c + 3d = 1. \qquad \text{(XXI-5)}$$

The formulas given in the literature for maximum pressure in a shock wave satisfy the requirements of similarity theory. For instance, for great distances, the formula

$$p_{max} = \text{const} \cdot \frac{\sqrt[3]{m}}{r} + p_0. \qquad \text{(XXI-6)}$$

is adopted. However, when processing experimental data, one for the momentum, frequently resorts to the formula

$$i = \text{const} \cdot \frac{\sqrt{m}}{r}. \qquad \text{(XXI-7)}$$

which contradicts Eq. (XXI-5). Such a deviation may depend on the nonobservance of similarity conditions when measuring the momentum, especially in the case of powerful charges and great distances. Vlasov [3] and Savich [113] give formulas for the momentum which are completely correct from the viewpoint of similarity theory.

It was noted above that one of the similarity conditions is the constancy of explosive density. Sadovskiy established experimentally that with $\frac{r}{\sqrt[3]{m}} > 1$ (m, kg) the parameters of a shock wave depend only on the weight of the charge but not on its density, in which case Eq. (XXI-4) rather than (XXI-2) holds. These experiments compared the effect of the explosion of pressed TNT and powder TNT of varying density (from 1.6 to 0.3), where decrease in pressure and momentum did not exceed 2 to 3%.

On the other hand, a low-density charge exploded in normal atmosphere can be regarded as being similar to a high-density charge exploded in air under high pressure.[54]

Under this assumption, Sadovskiy's results permit us to predict, with the aid of similarity theory, the dependence of the quantities characterizing the wave on the density of air. We give here, without their derivation, the final formulas in which air density is expressed by its pressure p_0 and temperature T_0

$$p_m = p_0 f\left(\frac{r\sqrt[3]{p_0}}{\sqrt[3]{mT_0}}\right); \; i = \sqrt[3]{mp_0^2 T_0}\; h\left(\frac{r\sqrt[3]{p_0}}{\sqrt[3]{mT_0}}\right) \qquad \text{(XXI-8)}$$

or, for power functions, the realtionship between the exponents of distance r, charge mass m, and atmospheric pressure p_0 and temperature T_0, which we do not regard as constant here,

$$p_m = \text{const} \cdot r^{-3b} m^b p_0^{1-b} T_0^b; \; i = \text{const}\, r^{1-3d} m^d p_0^{1-d} T_0^d.$$

Exaclty the same relationship between exponents can be obtained assuming that $p_m - p_0$ depends on the parameters according to the exponential law. At a great distance, the damping of pressure change amplitude and momentum is inversely proportional to the distance. Let us take here this limit law as a result of the experiment. In that case we get

$$p_m = p_0 + \text{const}\cdot r^{-1} m^{1/3} p_0^{2/3} T_0^{1/3};\ i = \text{const}\cdot r^{-1} m^{2/3} p_0^{1/3} T_0^{2/3}.$$

It would be interesting to study experimentally the problem of the effect of atmospheric conditions on the propagation of shock waves. A change in the temperature from +40 to -40°C changes $T_0^{1/3}$ by 10%, $T_0^{2/3}$ by 22%.

Although spherical propagation of shock waves is much more important, cylindric and one-dimensional propagation are also of some interest. Cylindric propagation occurs when a long charge explodes, and the shock wave is radiated at a distance from the charge which is less than the length of the charge. One-dimensional propagation occurs when a shock wave propagates in a pipe. The extremal laws derived above for spherical propagation can be readily changed for the latter two cases. Thus, in the one-dimensional case

$$p_{1,m} = p_0 f(r/m_1)\ i = p_0 m_1 f(r/m_1),$$

where m_1 is the mass of explosive per unit of cross section.

Motion at a short (or small as compared with the dimensions of the charge) distance from the surface of the charge can also be regarded as one-dimensional motion. In this case, however, particular care must be used on account of the dependence of the distribution of the pressure and motion of explosion products on the character of blast wave propagation, which is spherical with central priming of the charge, or plane with simultaneous priming along a plane parallel to the surface of the charge ([8], 2nd paper).

Simulation is particularly valuable when studying the propagation of waves under difficult or complex geometric conditions, for instance when studying various methods for protecting ventilation ducts from blast waves, the difraction laws of a shock wave at the obstacle, and so on [117]. It is obvious that in these cases it is essential to maintain similarity both in the position of the surfaces reflecting the shock waves and in the position of the measruing instruments. The results of measurements depend not only on the distance of the instrument from the charge, but also on its position with respect to the obstacles, etc.

Chapter 22

Simulation and Similarity of Destructions Caused by Shock Waves

Destruction occurs when the stress in a material reaches limit values. Similarity will therefore be achieved if we use the same material in the model as in the actual case, and, of course, if the model is geometrically similar to the actual object.

By using the same material we will be sure to have a similarity in the propagation of the shock wave, in its transition from one medium to the other, and so on. We have seen that the characteristic pressure amplitude is constant. In similar explosions the pressures are identical at similar points.

The regions in which the stresses caused by the explosion exceed the permissible values and bring about the destruction of the material will also be similar.

Destruction requires that a specific deformation be reached, i.e., that certain particles of the body be shifted with respect to other particles. Inertial forces and elasticity prevent deformation and destruction from occurring instantly. Could it be that the existence of a specific deformation time will lead to a violation of similarity?

But we can easily see that similarity will be maintained. It is precisely the inertia of the substance, which depends on density, and its elasticity that determine the speed of sound in the substance. It can be formally shown by means of analysis that from density and elasticity we can plot deformation time only on the basis of the dimensions of the body, and this will be the time required by the wave to pass through the body. The time will turn out to be proportional to the size of the body. If we change the scale, deformation time changes following the same law as the one governing the shock wave action time, and the relationship between the times will remain constant. This ensures similarity of the phenomena.

Similarity is also applicable to the more complex type of destruction, in which the shock wave momentum is decisive (see Chapter 20) rather than peak pressure.

Let us take an elastic beam, the oscillation period of which exceeds shock wave action time.

By reducing the dimensions of the charge, the beam and the distance between them by a factor of n, the oscillation period of the beam will decrease by a factor of n, and the frequency will increase by a factor of n. This can readily be verified with the aid of elasticity theory for any specidic practical method of securing the beam.

The mass has decreased by a factor of n^3, at a similar point the shock wave momentum per unit of surface has decreased by a factor of n on account of a decrease of the shock wave width and a decrease in shock wave action time at a constant peak pressure, and the surface receiving the pressure has decreased by a factor of n^2. Thus linear velocity reached by the beam as a result of the effect of pressure momentum will be independent of the size of the beam. The amplitude of the oscillations will be of the order of the product of velocity × period, i.e., it will be proportional to the size of the beam. Hence we see that the relative deformation and density of elastic energy proportional to the square of initial velocity are identical in the model and in actuality. The result will also be identical, namely, the presence or the absence of destruction. Let us note that similarity will not be violated by friction which depends on velocity and on the load in the case when the load is also assigned in the fundamental shock wave action, since velocity and pressure are the same in similar systems.

A less trivial case is the one frequently encountered in structural mechanics. It is the case in which the stability of the structure and the effort required for its destruction depend on the structure's weight. The simplest instance of this kind is the sandy area without cohesion. Another instance is a stack of bricks, the solidity of which depends on the weight of the bricks and on the friction produced by the pressure of one brick on the other. Khariton emphasizes that such a type of stabiltiy very frequently determines the resistance of a structure to destruction. The stack of bricks represents one extreme example in which the weight determines internal cohesion. A solid steel box, which is easier to topple as a whole than to destory, is another extreme example in which the explosion works against the force of gravity.

Here the impossibility of a strict similarity is obvious. The theory now includes acceleration of gravity g expressed in terms of length/time^2. Together with the characteristic velocities of the explosion process, e.g., c_0, the presence of g permits the plotting of the length, e.g., c_0^2/g and time c_0/g. The absence of similarity is obvious: if we compare two charges of different size buried in the sand at an appropriate depth, we can see that the pressure of the soil at the level of the charge is proportional to the depth, and to the size of the charge. Likewise, minimum pressure required for the toppling of a wall in the second example is also proportional to the size. However, atmospheric pressure and blast pressure do not depend on the size.

Thus, with a change in size there is also a change in the ratio of soil pressure or the pressure required for the beginning of destruction to blast pressure, and similarity is therefore violated.

An excellent simulation method was proposed by Pokrovskiy [109]. To obtain similarity as we change the scale of the experiment, we must also change the length proportionality. Pokrovskiy obtains this by changing acceleration, and replacing gravity with centrifugal force. The model is exploded on a centrifuge and the dimensions are reduced with respect to nature at the same ratio of centripetal acceleration to acceleration of gravity. We can readily verify that soil pressure at similar depths will be similar.

Pokrovskiy made extensive use of his method for the purpose of modeling large-scale explosions for excavation, and also for the purpose of studying the effect of various soils and different positions of the charge on the result of explosions. The linear modeling scale in his experiments reached 29, i.e., all the dimensions of the model were reduced by a factor of 29 as compared with the dimensions of the real object. The weight of the charge, which characterized the cost of the experiment, was reduced by a factor of 25,000.

Zel'dovich and Khariton proposed an approximate method for simulating the work of explosives against the forces of gravity. It is based on the fact that the new criterion on which depends the absence of similarity in the case of a change in scale, differs appreciably from unity. Thus, if we write this criterion as a ratio of characteristic length c_0^2/g to the size of the charge R, then for a charge weighing 1 kg we get $c_0^2/gR = 2 \times 10^5$. The ratio of static soil pressure to blast pressure yields, at a crater depth of several meters, a quanity of the order of $10^{-4} - 10^{-5}$. Thus, the criterion in the most varied formulations turns out to be sharply different from unity. This means that we are dealing here with the case in which not all the quantities are of the same order. It is obvious that we find ourselves in the domain of extreme or critical laws, in a domain, that is, in which we may expect self-simulation in the same way as self-simulation arises in hydrodynamics at very high or very low Reynolds Numbers.

We now have to find the physical nature of this self-simulation.

Let us give a closer look to the toppling of a wall (see Fig. 56). At the beginning of the preceding chapter we brought it up as an instance for a process which lasts considerably longer than the action of the shock wave (in this case the time ratio yields another criterion which sharply differs from unity), i.e., a process in which the decisive role is played by the general wave momentum. We divided the process into two stages: 1) the action of the wave on the object which determines its momentum, and 2) the motion of the object by inertia, which overcomes the force of gravity, and we readily find the conditions for similarity.

In fact, the object's momentum K, equal to the force momentum, (for a geometrically similar change of the system, in which the dimensions of the object and the distance between the object and the charge change proportionally to the dimension of charge R) is proportional to

$$K \cdot Fi \sim R^2 \cdot \frac{\rho_0}{c_0} R \sim \frac{\rho_0}{c_0} R, \qquad \text{(XXII-1)}$$

where F is the area on which the wave acts, i is the pressure momentum per unit of surface. The momentum of the object sufficient for its toppling will be determined as follows. The object's kinetic energy is equated to the work required for lifting the gravity center of the object to a height proportional to the size of the object,

$$E \sim \frac{K^2}{M} \sim MgR. \tag{XXII-2}$$

Into K of Eq. (XXII-1) we substitute the expression of the object's mass M by the characteristic dimension R and the object's density φ, and get

$$M \sim \varrho R^3;\ \frac{p_0^2}{c_0^2}\frac{(R^3)^2}{\varrho R^3} \sim \varrho R^3 gR;\ \frac{c_0^2}{p_0^2}\varrho^2 gR = \text{idem} \tag{XXII-3}$$

The sign idem adopted in similarity theory signifies that similarity will take place if the term on the left remains constant. For all explosions in the air under normal conditions c_0 = const, p_0 = const, the criterion is simplified and $\varrho^2 gR = \text{idem}$.

This criterion also cincludes exact simulation — the change in accleration g is inversely proportional to the size R (centrifugal simulation). But on the basis of the approximations made earlier we obtained a criterion which also admits another solution: the change in density is inversely proportional to the root of its dimensions. This method was proposed by Khariton and this author [105]. This method allows for a sufficiently wide change in the scale. By substituting a material with density 2 (stone) with a material with density 11 (lead) it becomes possible to reduce R by a factor of 30, which corresponds to the reduction of the charge by a factor of 27,000, i.e., it is possible to simulate the explosion of 1 ton of explosive by the explosion of 40 g of the same substance.

In Khariton's many experiments, the edgewise standing bricks turned out to be convenient indices for the distance at which the momentum of a shock wave drops to a specific value.

It is obvious that centrifugal simulation is necessary in more complex cases in which, along with a rigid structure, the soil also plays a role. The approximate simulation by changing density, as proposed by Khariton and this author, is considerably narrower in scope and the advantage of this method is only the simplicity of experimentation.

Chapter 23

Phenomena Occurring in the Immediate Vicinity of the Charge

Similarity theory makes it possible to reduce the relation between the quantities that characterize the effect of an explosion and the charge mass and the distance to two dimensionless functions with one dimensionless variable. By determining the form of these functions we will get a clear idea of the explosion phenomenon and the ensuing propagation of the shock wave. Here we are not going to study the explosion proper, i.e., the propagation of the blast wave along the explosive accomapnied by a chemical reaction which transforms the explosive into an explosion product or products. Our investigation will begin when the blast wave reaches the surface of the charge. We assume that the wall of the charge is very thin and hence ignore its effect. At a given instant of time the following will be abutting: on the one hand the unperturbed air or the material to be destroyed, and on the other hand the explosion products which have just been formed as a result of chemical reaction.

Computations relating to degeneration theory show that these explosion products move in the direction of the propagation of the blast wave. Their density is higher than mean density of explosion products so that pressure is twice as high as mean pressure. If the explosive is bordering on the obstacle, then at the instant when the blast wave reaches the boundary, the moving explosion products collide with the obstacle and are abruptly inhibited or stopped. At the pressures with which we are dealing, any material is plastic. The velocity acquired by the material of the obstacle under the effect of the explosive products is bounded not so much by the strength of the material as by its inertia, i.e., density and compressibility (the latter determines the velocity at which the disturbance propagates and, hence, the amount of material involved in the motion per unit of time).

When explosion products hit steel or iron (density 7.8, whereas the explosion products density does not exceed 2.5) we can say that the motion of the explosion products is virtually stopped. At this instant a shock wave begins to move from the boundary into the charge, which stops and compresses the explosion products. Qualitatively this phenomenon is analogous to the reflection of a shock wave (Chapter 19). Quantitatively there is a certain difference, and computations show that the pressure of the explosion products increases approximately twice when the shock wave hits an obstacle.

If the explosive is of low density, and if the explosion products can be considered and ideal gas, then in the shock wave front the velocity of the explosion products amounts to about 45% of detonation velocity, density in the wave front attains 180% of the initial one, and the temperature rises 10% as compared to mean temperature. Pressure therefore increases by a factor of 2 as compared with mean pressure $\overline{p}$ or the pressure which is developed by a slow adiabiatic reaction of an explosive with constant volume. Ixmaylov showed that this pressure is almost tripled (and thus reaches 5-6 $\overline{p}$) when the explosion products are slowed down by an absolutely hard (rigid) obstacle placed in the path of the blast wave. However, the explosion of commerical explosives deviates considerably from ideal conditions. The ratio of explosion product velocity to blast velocity decreases. The ratio of explosion product density in the wave front to the mean density of explosion products also decreases. But, at the same time, the compressibility of explosion products also decreases. An identical change in density causes a change in pressure greater than in an ideal gas; sound velocity also increases; hence the impact against the obstacle becomes harder. The pressure ratio, of mean p, maximum p_{det} in a detonation wave, the pressure of reflection of a shock wave by a rigid obstacle p_{refl} found for an ideal gas, changes somewhat in dense explosion products with considerable deviations from ideal conditions.

Table 5 shows the fundamental constants for some characteristic explosion products. These are explosion heat Q kcal/kg, the volume taken up by the explosion products under normal conditions (0°C, 1 atmosphere), V_0 liter/kg; explosion product temperature in the blast wave front T_{id}°K, detonation velocity at low density D_{id}m/sec, and explosion product velocity in the wave front u_{id} m/sec computed according to detonation theory without taking into account any deviation from ideal conditions, initial density of explosives φ_0 g/cm^3 or kg/liter, and detonation velocity D m/sec measured at this density. The difference between D and D_{id} characterizes the deviation of the state of the explosion products from ideal conditions. In the following columns we have computed the density of explosion products φ and explosion product velocity in the direction of the propagation of the wave u. P_{det} is the pressure of explosion products in the detonation wave computed considering the deviation from ideal conditions and compression of explosion products in the wave. The column p_{refl} shows the pressure developed by an abrupt declaration of explosion products, whereas their velocity and state are determined in the preceding columns.

Table 5

	Q	V_0	$T_{ид}$ g)	$D_{ид}$ g)	$u_{ид}$ g)	ρ_0
Тротил a)	1085	685	3630	1930	890	1.59
Нитропентаэритрит b)	1530	768	5000	2400	1090	1.60
Нитроглицерин c)	1517	716	5200	2360	1080	1.60
Азид свинца d)	200[1] 325[2]	270[1] 230[2]	2800	1250	570	4.70

	D	ρ	u	$P_{дет}$ e)	$P_{отр}$ f)
Тротил a)	6900	2.10	1700	190 000	430 000
Нитропентаэритрит b)	7900	2.12	2000	250 000	560 000
Нитроглицерин c)	7900	2.12	2000	250 000	560 000
Азид свинца d)	5890	6.30	1500	450 000	900 000

[1]Computed for lead in vapor form.

[2]Computed for liquid lead.

CODE: a) TNT, b) nitropentaerythritol; c) nitroglycerine;
d) lead azide; e) det; f) refl; g) id.

Computations based on detonation theory (considering ideal explosion products), were performed by Dautriche [119], Schmidt [124], and Vlasov [3]. The computations were based on the assumption that we can apply the equation of state to explosion products

$$p = \frac{RT}{v - b}$$

with constant b, or a vlaue of b that depends on specific volume v (Schmidt). Landau showed that in reality this equation of state is not applicable to the density attained in explosion products. Molecules cannot be considered incompressible. In the first approximation explosion product pressure depends on the density of explosion products (proportional to the cube of the density), but does not depend on temperature. Landau's and Stanyukovich's computations [107], performed in 1944, show that the measured detonation rate corresponds to a smaller specific volume and a higher pressure as compared with earlier computations. Khariton noted that the equation of state adopted by Landau requires an appreciable amount of blast energy to perform the compression of explosion products, and the temperature of explosion products (with a high initial density of the explosive) is considerably lower than the one given in the table under T_{id}.

The structure of a detonation wave is characterized by the fact that at the instant it is formed the explosion products have maximum density, velocity and pressure. Behind the wave front there follows a more or less rapid deceleration and expansion of explosion products [8, 108]. All the values for pressure given here are referred to the wave crest. Immediately after the collision between the wave and the obstacle, i.e., after a tremendous pressure p_{refl} has been developed, pressure begins to drop quite rapidly. Below, when we study the pressure momentum of an explosive, we shall see how the time during which pressure drops is determined. In order to magnitude this time is equal to $R \times 10^{-6}$ sec, if R is the effective radius of the charge expressed in centimeters. For a charge of 1 kg this time is of the order of 5×10^{-6} sec.

To compute the time we juxtapose the force momentum and maximum pressure. Let us imagine a charge of 1 kg TNT in the form of a cylinder 10 cm in diameter and 8 cm high. The area of the cylinder base is 80 cm^2. Assuming maximum pressure developed at the reflection of the wave to be 430,000 atmospheres, we get the maximum force that acts on the obstacle on which the charge is placed, namely, 3.5×10^7 kg.

The momentum value of 100 kg x sec/kg found experimentally (Kudryavtsev's experiments, quoted here from Sadovskiy) corresponds to the effective action time of the force computed above, namely, 3×10^{-6} sec. For sound velocity in explosion products of the order of 5×10^{5} cm/sec (we find this value from the measured detonation rate) the time during which an expansion wave covers a distance of 5 cm amounts of 10×10^{-6} sec. It is obvious that in reality pressure drops gradually and attains atmospheric values during a considerably longer time. The quantities 3×10^{-6} or 10×10^{-6} sec are only effective values, i.e., they are the time during which pressure drops several times.

What happens when the detonation wave reaches the free charge surface which borders on the air? When the explosive is exhausted, the incandescent explosion products (in motion and under high pressure) are in contact with the unperturbed air. The surface of the charge becomes the surface of pressure discontinuity, and of the discontinuity of velocity and gas tmeperature (Fig. 57). Thus, we are dealing here with the problem discussed in Chapter 16. The expanded and accelerated explosion products speed forward in the direction in which the blast wave propagated, pushing the air in front of them and compressing it (Fig. 58). The motion of the boundary of the expanded explosion products and of the compressed air is determined from the condition of pressure equality on both sides of this boundary. The only new element as compared with Chapter 16 is the fact that on the discontinuity surface there also occurs a change in chemical composition (explosion products — air). As the discontinuity propagates, the surface of the change of composition coincides identically with the surface of the discontinuity of special kind on which there occurs the change in temperature and entropy without changes in pressure and velocity. All the results of Chapter 16 remain valid.

Emile Jouguet [120] applied the theory of propagation of an arbitrary discontinuity to the computation of a shock wave arising on the surface of a detonating explosive. He performed his computations in connection with the experiments carried out by Perrota and

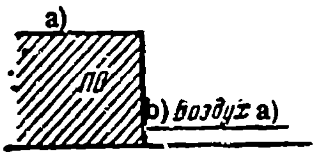

Fig. 57.

CODE: a) Explosion products; b) air.

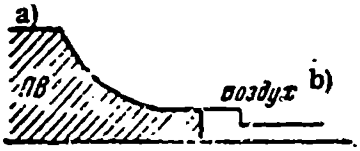

Fig. 58.

CODE: a) Explosion products; b) air.

Gawthrop [122]. The same problem was studied later by Landau and Stanyukovich [108].

The results agree with the experimental data, in particular in the problem concerning the effect of the composition of the atmosphere surrounding the charge on the expansion rate. The velocity obtained by the shock wave in the air and the velocity of the interface between explosion products and the air are very high and may exceed the detonation rate of the explosive with which they are not directly connected. In correspondence with the high velocity of the explosion products that compress the air, there occurs in the shock wave a pressure which is high compared with atmospheric pressure, but which amounts to a minor portion of the initial pressure of the explosion products. If the air is enclosed as a thin layer between the explosive charge and the obstacle, then the shock wave, once it reaches the surface of the obstacle, will be reflected and will change its direction. When the reflected wave will reach the interface between the explosion products and the air, there will be a partial passage through the explosion products, and so on. The layer of air between the explosive and the obstacle delays the increase in pressure acting on the wall and, hence, delays the instant when maximum pressure is attained. If the explosion products were able to exert continuous pressure, the presence of the layer of air would not change the final pressure exerted on the wall, as a soft pad does not reduce the pressure of a load on the base. In reality, the structure of a detonation wave determines a rapid pressure drop which depends on the levelling of the pressure and the expansion of explosion products in a direction opposite to the direction of detonation propagation (i.e., toward the center of the charge). The delay in the transmission of explosion product pressure to the obstacle (due to the presence of the air gap) results in that during that delay the explosion product pressure drops and maximum pressure acting on the obstacle also decreases. The extremely sharp dependence of the disruptive force of an explosion on the distance between the charge and the armor plate is well-known.

The instructions of the engineering corps for demolition work (Voyenizdat NKO [National Commisariat of Defense], 1941) gives the following rule. To penetrate a steel sheet, the weight of the charge must be given in terms of 25 grams of normal explosive per 1 cm^2 of the cross section to be penetrated. The cross section is computed as the product of the length of the line along which penetration occurs, and the thickness of the sheet.

In the case of an air gap between armor plate and charge, or if the armor plate consists of steel sheets with air gaps in between, the "Instructions" require that the air gap be added to the calculated thickness of the sheet.

Thus, according to the rule (which, of course, is approximate) one must conclude that a charge which, for example, can penetrate a 5 cm thick steel sheet if tightly attached to it, will penetrate an armor plate only 3 cm thick if it is placed at a distance of 2 cm from it.

The pattern will be different if at some distance from the charge there is a body the dimensions of which are small as compared with the distance from the charge. Such a body will first be subjected to the effect of an air shock wave. Soon after the reflection of the shock wave by the surface of the body, the shock wave travels further around the body. After this the force acting on the body becomes the drag of the body in a flow of air compressed by the shock wave, and depends on the density and air velocity and the resistivity factor of the body. Then the interface between the explosion product reaches the body, and further on the body is flowed around by the expanded explosion products rather than by air. On the interface the explosion product pressure does not differ from air pressure. How does the force change that acts upon the body? To answer this question we must compare the density of explosion products and that of air.

The expansion of explosion products occurs isentropically and is accompanied by a temperature drop in the explosion products. The compression of air by the shock wave following Hugoniot's adiabatic curve causes an increase in temperature.

A rough, approximate computation performed for TNT (for the initial data see table) detonated in the air yields the following results. The velocity of expanded explosion products, equal to the velocity of compressed air, is 4700 m/sec; the pressure of expanded explosion products and of compressed air is 250 atmospheres. The temperature of explosion products drops to 1100°K (830°C), the density of explosion products is 0.1 g/cm^3; in the air the shock wave propagates at a velocity of 5250 m/sec, air temperature reaches 7600°K and density is 0.012 g/cm^3. The absolute value of wave velocity agrees sufficiently well with the data of Perrota and Gawthrop, who recorded a wave velocity of 4600 m/sec in air, and 5560 m/sec in hydrogen in the case of a weaker explosive (density 1.32, detonation rate 4600 m/sec).[55]

Let us note that as a result of the expansion of explosion products there occurs a temperature inversion: the temperature of compressed air turns out to be considerably higher than that of the explosion products in contact with it. There is no contradiction with the principles of thermodynamics here. We have only isentropic processes (expansion) and these are accompanied by entropy increase (compression in the shock wave). The first prinicple also is not violated: the amount of air compressed in a unit of time is considerably smaller than the amount of expanding explosion products.

The molecular weight of air and the explosion products in the case of explosion of organic substances only differ slightly from one another. With equal pressure, the density ratio is inverse to the temperature ratio.[56] The density of expanded explosion products is considerably greater than the density of compressed air. The force acting on the body grows approximately proportionally to the density at the instant when the explosion products expand and reach the body. At the same time (and this seems surprising) the thermal effect on the body's surface also increases. The temperature of explosion products is lower than the temperature of compressed air, but in the case of supersonic flow one must take into account both the thermal and the kinetic energy of the moving gas (see Chapter 4, "temperature at rest").

The velocity of explosion products and of the air are identical, and the conversion of kinetic energy into thermal energy increases the temperature by an identical quantity. Thus, the difference in the "temperature at rest" of the air and of explosion products is relatively smaller than the difference in the true temperatures of the air and of explosion products. In the example given above, where the true temperatures of air (7600°) and of explosion products (1100°) are in a 7 : 1 ratio, the temperature at rest of air (24000°) and of explosion products (17000°) are in a 1.4 : 1 ratio.[57] The intensity of the thermal effect depends not only on the temperature of the gas surrounding the body, but also on other factors which determine the intensity of the heat flow. In the case under study, the heat flow and the thermal effect increase on account of an explosion product density increased eight-fold as compared with the density of air.

Experiments confirming the above were performed by Michel-Levy and Muraour [121] in 1934-1936. They studied the problem of the nature of the luminescence of the explosion of lead azide crystals. Photographs show that this luminescence is particularly intense where the shock waves collide. The intensity of luminescence and its spectrum depend essentially on the atmosphere surrounding the crystals. The most intense luminescence occurs in argon, and the least intensive one in butane, in accordance with the thermal capacity of the substances. With a given gas composition (argon), increased pressure reduces the intensity of luminescence in accordance with the change in shock wave amplitude. An exquistie experiment is the one in which a metal (barium) was introduced into the explosive and the gas. When a barium compound was added to an azide charge, barium lines could not be detected in the luminescence spectrum. In other experiments, barium was introduced into the gaseous phase by burning prior to the experiment a small amount of a pyrotechnic compound which yields a finely divided, slowly settling smoke that contains barium oxide and carbonate. In the latter case, after explosion, the luminescence spectrum abunded with barium lines. Together with excited barium atoms, the spectrum reveals the existence of excited barium ions, and thus reminds us of a spark spectrum rather than an arc spectrum. Michel-Levy's and Muraour's tests show that at blast in the atmosphere surrounding the explosive there arise shock waves of a wide amplitude which heat the gas to tremendous temperatures, exceeding many times the temperature of explosion products. These temperatures are particularly high owing to the low thermal capacity of argon. All the facts observed agree with this.

In a theoretical paper, Jouguet [120] compares the propagation of shock waves from an explosion in gases of different molecular weight (hydrogen, air, and carbon dioxide). The velocity of waves caused in various me .ia is in good agreement with test data. Jouguet does not perform a direct computation of absolute values, and thus avoids the problem of detonation theory of explosives and the state of explosion products of high density. Instead, Jouguet uses the velocity of a shock wave in the air to characterize the state of explosion products, and from there computes the velocity of waves in other media.

Vlasov [3] goes many steps further. He overcomes great difficulties which depend on the fact that he is dealing with non-ideal media, and computes the parameters of a shock wave in air and its velocity. His results are in good agreement with tests data (nitromannite: Vlasov computed 6100 m/sec, and Byurlo observed 6430 m/sec).[58] To characterize the state of explosion products Vlasov uses the measured detonation rate.

We must also mention here the extremely interesting and exhaustive computations performed by A. A. Grib on the surface of explosives, contained in his dissertation [102] (Leningrad Mining Institute, 1940). The problem is solved under the assumption of a distribution of pressure and motion which correspond to an instant chemical reaction of the entire explosive, condensed or gaseous.

Finally, let us dwell on the second possibility of interpreting the explosive momentum. According to computations mentioned above, a charge weighing m kg placed on a surface, dovelops at blast at a force momentum I = 100 m kg/sec. According to Chapter 18, such a momentum corresponds to a mean value of the velocity component of explosion products, a normal surface $\bar{u}_n = 100\ g = 1000$ m/sec.

The force momentum turns out to be one-half to one-third of the force momentum developed by an ordered outflow of explosion products from a Laval nozzle of a jet engine in which all the explosion products move in one direction. We can readily see that at the explosion of an open charge the explosion products expand uniformly in all directions of the hemisphere. We denote mean velocity in the radial direction by u_r, and find $u_r = 2u_n =$ 2000 m/sec. Half the momentum is lost as a result of the expansion of explosion products not only in the direction normal to the wall but also in other directions.

Chapter 24

Laws Governing the Propagation of a Shock Wave at a Great Distance from the Charge

In the preceding chapter we studied the phenomena that occur in the immediate vicinity of the charge. For the quantitative estimates we proceeded from the idea that detonation theory determines the state of explosion products in the blast wave front independently of the shape of the charge, the position of the primer and similar factors. All these factors are very important for the pressure distribution. Owing to the fact, however, that detonation rate is exactly equal to the rate of disturbance propagation along the explosion products, these factors do not effect the detonation rate and the state in the wave front.

However, after the first contact between explosion products and the air (or the material to be destroyed) the motion will be affected by pressure distribution in the deeper layers of the explosion products. To determine the motion at this stage requires extremely laborious and complex computations, all the less attractive since the result is different for each case.

Only at the next stage can we expect that at a sufficient distance from the charge the dependence on the actual geometry of the explosion will subside and a specific form of the shock wave will appear which depends only on the total amount of explosive but not on specific features of the given charge such as the position of the primer or the presence of a shell, or the shape of the charge, which are extremely important at a close distance. The condition imperative for the formation of such a steady wave form lies in the fact that motion involves a certain amount of air that must exceed the amount of explosive by at least several times.

When energy is transferred from the explosion products to the nearest layer of air, and from that layer to the next one, and so on, the wave becomes independent of the peculiar features specific for each single charge. We may expect the existence of two limit regions in accordance with the simplifications to which the laws of shock wave theory are subjected in two limit cases; 1) powerful shock waves, $p \gg p_0$, and 2) weak shock waves $p - p_0 \ll p_0$ which qualitiatively approach the characteristics of sound (see Chapter 3).

In the first case, according to Landau, there will be a transition to limit if we neglect p_0 with respect to p. We may obviously disregard in this case the initial temperature and energy of air with respect to its temperature and energy after compression by the shock wave. In such an approximation the distribution of pressure and temperature changes in time but remains similar to itself.

The critical laws of powerful shock waves provide for a constant relationship between kinetic and thermal energy of the compressed substance. Total energy of all the substance involved in the motion is also constant during the motion time. In the case of the above simplifications, the involvment of new layers of air is not accompanied by any appreciable increase in total energy which is read from the absolute temperature zero.

Mean energy density drops inversely proportionally to the volume covered by the wave, i.e., inversely proportionally to the third power of the path travelled by the wave. In the case of similar distribution, the local values of energy density drop in the same fashion. According to the laws of an ideal gas with constant thermal capacity, pressure depends only on energy density ϵ, but not on the density of substance ϱ

$$P = \frac{RT}{v} = \varrho RT = (k-1)\varrho c_v T = (k-1)\varepsilon, \qquad \text{(XXIV-1)}$$

where R is the gas constant, lower R is the charge radius, r is the distance from the charge center.

Thus, in the extreme case mentioned above, Landau arrives at the following simple formulas

$$\bar{p} = \frac{(k-1)QM_1}{4\pi/3 \cdot r^3}; \quad M = \frac{4\pi}{3} \cdot r^3 \varrho_0; \quad T = \frac{QM_1}{c_v M} \approx T_1 \cdot \frac{M_1}{M} \qquad \text{(XXIV-2)}$$

where $\bar{p}$ and $\bar{T}$ are mean pressure and temperature, Q is explosion heat of the explosive, M_1 is charge mass, M is the mass of air involved in the motion, ρ_0 is initial air density. In reality, however, there is hardly a region in which this extreme law is strictly applied. For it to the applied, the follwoing two conditions must be satisfied at the same time

$$\frac{M}{M_1} \gg 1; \quad \frac{\bar{T}}{T_0} \gg 1. \tag{XXIV-3}$$

According to the formulas mentioned above

$$\frac{M}{M_1} \cdot \frac{\bar{T}}{T_0} = \frac{T_{expl}}{T_0} \tag{XXIV-4}$$

However, for explosion products and air at room temperature the ratio T_{expl}/T_0 does not exceed 10-15. The entire interval from $\frac{M}{M_1} = 0_{expl}\ \frac{\bar{T}}{T_0} = 2$ is covered while the shock wave radius changes by 2 to 2.5 times.

In reality, however, for a small r we must take into account the effect of the initial distribution of pressure and density in explosion products. The ratio M/M_1 (the mass of the air involved in the motion to the mass of explosion products) reaches unity at a value $\frac{r}{\sqrt[3]{M}} = 0.6 \frac{m}{\sqrt[3]{kg}}$, i.e., at a distance equal to 11 charge radii. This same quantity gives the distance of the direct effect of explosion products on the obstacle. However, already at $\frac{r}{\sqrt[3]{M}} = 1.5$, at a distance equal to 27 charge radii, the amount of heat introduced by the air involved in the motion becomes equal to the explosion energy (all figures are given for typical explosives).

Mean pressure at this instant is twice that computed by the limit formula according to which the drop of p is inversely proportional to r_0^3. The descrepancy increases further on. Vlasov [3] feels that there is a good agreement between experimental data and the formula for pressure on the obstacle normal to the direction of wave propagation

$$p_{max} = 250\,000 \left(\frac{r}{R}\right)^{-2.6} = 120 \frac{M^{0.87}}{r^{2.6}} \left(m,\ kg,\ \frac{kg}{cm^2}\right), \tag{XXIV-5}$$

which he applies to the entire interval from r/R = 1 (pressure on the body in contact with the explosive) to r/R = 100. The theoretical conclusion of this formula is not convincing. It is impossible to descirbe with one single formula all the various different processes which depend on different factors (non-ideal condition of explosion products with r/R close to 1, effect of explosion products with r/R up to 10, a powerful shock wave with r/R from 10 to 100, etc.). It must be noted that in the interval between computed values for r/R = 1 and systematic measurements beginning with $\frac{r}{R} > 15$, there is only one experimental point.

Thus, if we take Vlasov's formula to be empirical, then we cannot consider it verified in the entire interval for which it is recommended. We must admit, however, that in the interval in which measurements are made, their agreement with Vlasov's formula is satisfactory, whence the formula's practical applicability.

Let us now study the second extreme case, the propagation of a blast wave at a considerable distance from the charge, where its amplitude is small. At limit the propagation laws must, obviously, coincide with acoustic laws with which we already familiarized ourselves at the beginning of this monograph (Chapter 3). The acoustic laws provide for the propagation of a wave with an amplitude constant in the linear case and dropping, as 1/r, in the spherical case, but without change in the wave width and form. Consequently, acoustic laws cannot be used to determine the form and the width of a wave even in the first approximation. Hence in the following we will have to pay particular attention of the deviations from acoustic laws, which decrease as the amplitude drops, and to experimental data regarding the amplitude and form of blast waves.

Figure 59 shows the curves of pressure change with time at different distances from the explosive charge, taken from the paper by Bernal' [101]. We note that the unperturbed air is subjected initially to a sharp compression which is followed by a pressure drop that passes through a minimum and returns to atmospheric values. Obviously, the instant distribution of pressure in space reminds us of the curves of pressure change with time, 2 to 3 milliseconds corresponding to a wave width of about 1 m.

Thus, the front of a blast wave represents a shock wave which is followed by a rarefaction (expansion) wave. To predict the law of blast wave change, we have to remember the kinematic and thermodynamic relationships between a shock wave and a continuous expansion wave.

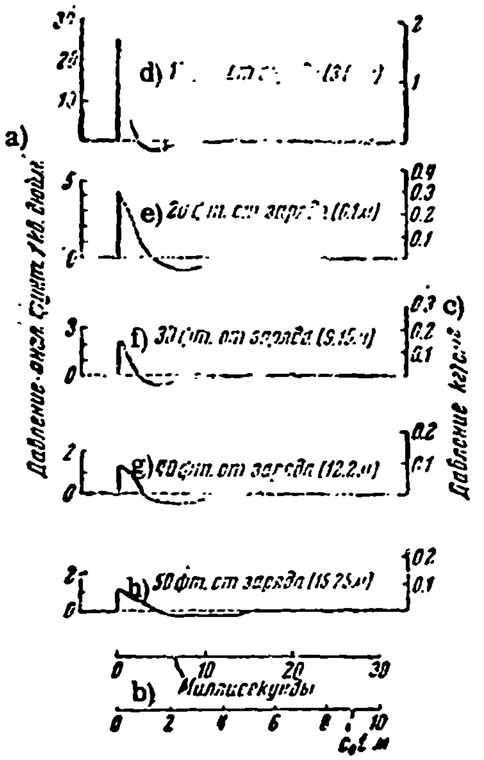

Fig. 59.

CODE: a) Pressure, feet/square inches; b) milliseconds; c) pressure, kg/cm^2; d) 10 feet from charge (3.0 m); e) 20 feet from charge (6.1 m); f) 30 feet from charge (9.15 m); g) 40 feet from charge (12.2 m); h) 50 feet from charge (15.25m).

In a continuous wave in which neighboring states differ infinitesimally, each propagates in space at a velocity equal to the sum of sound velocity and substance velocity.

The velocity of shock wave propagation is less than the sums of motion velocity and sound velocity in the substance compressed by the wave within the region covered by motion. Pressure drop inside the regions through which the wave has passed is transmitted to the shock wave surface and weakens the wave. Hence the amplitude of a shock wave drops faster than drops the amplitude of a weak sound wave.

Another peculiar feature of the shock wave propagation investigated here consists in the fact that entropy changes with shock compression. As a consequence, after the passage of the wave the air does not return to a state equivalent or identical to its initial state (prior to the disturbance).

In an acoustic wave, the energy of wave motion is fully transmitted from the layers involved earlier in the disturbance, to the layers which are involved in the motion as the wave propagates. In the case of a shock wave, a part of the energy of wave motion gets stuck forever in those layers through which the wave has passed, where it is irreversibly consumed for their heating. This circumstance causes a gradual decrease in the energy of wave motion in the case of a shock wave, and it also causes a drop in shock wave amplitude under conditions in which the amplitude of an acoustic wave is constant or increases the drop in the amplitude of a shock wave as compared with that of an acoustic wave under conditions in which the amplitude of an acoustic wave drops.

Finally, the need for the expansion of a wave of finite amplitude can be seen immediately.

Let us call a "wave" as before the entire region covered by the disturbance in which velocity and excess pressure (as compared with atmospheric) are different from zero. The front edge (with respect to the direction) of the wave represents a shock wave that compresses the air. The velocity of this wave is greater than sound velocity in unperturbed air. The back edge of the wave represents either a continuous wave (as in Fig. 59) or a shock wave which returns the gas to its initial state.[59] The velocity of the back edge is equal to or smaller than sound velocity in air in its initial state. Consequently, the front edge of the wave moves faster, which leads in time to an increase in the distance between the front and the back edge of the wave, i.e., to an increase in the width of the wave.

In Chapter 11 we have proven in detail and in general the reciprocal connection of the three peculiar features: the fact that shock wave velocity is greater than sound velocity at the initial state; the fact that shock wave velocity is less than sound velocity in a compressed gas; the fact that the passage of a shock wave is accompanied by an increase in entropy, i.e., by an irreversible conversion of energy into heat.

In view of the fact that these three characteristics are very closely connected, it is natural that the use of any one among them to determine the law of the change in amplitude and width of a wave as it propagates must lead to identical results.

Before studying spherical propagation which interests us because of its association with the theory of explosives, we shall look into the simpler case of linear propagation.

Linear (one-dimensional) motion occurs when a gas moves through a straight pipe with a constant cross section. In studying this case we ignore the losses due to the interaction of the gas with the lateral walls of the pipe.

Crussard [118] established for the first time in 1918 the limit law of such a motion.

According to Crussard, we study a triangular wave shown in Fig. 60. As time goes by the distance between each pair of points a, b, which correspond to different pressures, increases so that propagation speed (equal to the sum of gas velocity and sound velocity) increases as pressure increases. As a whole, the wave represents a totality of shock wave U in which there occurs a rapid compression, and an expansion wave UP following it, in which gas pressure drops.

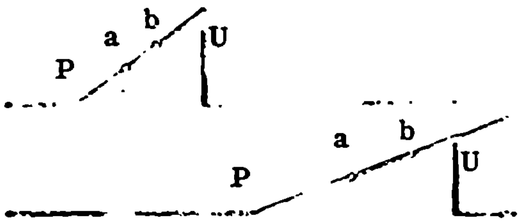

Fig. 60.

We write the equation of propagation of state a

$$x_a = x_{a0} + (c_a + u_a)\, t. \qquad \text{(XXIV-6)}$$

According to the laws of acoustics

$$c = c_0\left(1 + \frac{k-1}{2k}\cdot\frac{\Delta p}{p_0}\right);\quad u = c_0\frac{1}{k}\,\frac{\Delta p}{p_0}; \qquad \text{(XXIV-7)}$$

we write $\frac{\Delta p}{p_0} = \pi$, and find

$$x_a = x_{a0} + c_0 t + \frac{k+1}{2k}\, c_0 \pi_a\, t. \qquad \text{(XXIV-8)}$$

If at the initial instant there was a linear distribution of pressure depending on the coordinate, then it also remains linear later.[60]

$$t=0,\ x>x_{00},\ \pi=\frac{1}{\alpha}(x-x_{00});\ x<x_{00},\ \pi=0; \tag{XXIV-9}$$

$$x(\pi, t)=x_0(\pi)+c_0t+\frac{k+1}{2k}c_0\pi t=x_{00}+\alpha\pi+c_0t+\frac{k+1}{2k}c_0\pi t; \tag{XXIV-10}$$

$$\pi=\frac{x-x_{00}-c_0t}{\alpha+\frac{k+1}{2k}c_0t}. \tag{XXIV-11}$$

Given an initial linear distribution, Eq. (XXIV-9), at the instant t = 0, we obtain at an arbitrary instant t also a linear pressure distribution, Eq. (XXIV-11).

The velocity of shock wave D, the amplitude of which we denote by π^*, is equal to the arithmetic mean of c_0 and propagation velocity c + u of the state obtained after compression but prior to pressure π^*.

$$D=\frac{c_0+(c+u)}{2}=c_0\frac{1+1+\frac{k+1}{2k}\pi^*}{2}=c_0\left(1+\frac{k+1}{4k}\pi^*\right). \tag{XXIV-12}$$

We write the expression for the change in the amplitude in the shock wave as it propagates

$$\frac{d\pi^*}{dt}=\frac{\partial\pi}{\partial t}+D\frac{\partial\pi}{\partial x}. \tag{XXIV-13}$$

The expression differs from zero on account of the fact that D differs from c + u. Using the expression $\pi(x, t)$, Eq. (XXIV-11), we find

$$\frac{d\pi^*}{dt}=-\frac{k+1}{4k}c_0\frac{\pi^*}{\alpha+\frac{k+1}{2k}c_0t}, \tag{XXIV-14}$$

$$\ln\pi^*=-\frac{1}{2}\ln\left(\alpha+\frac{k+1}{2k}c_0t\right)+\text{const}, \tag{XXIV-15}$$

$$\pi^* = \frac{A}{\sqrt{\alpha + \frac{k+1}{2k} c_0 t}}, \qquad \text{(XXIV-16)}$$

where A is an intergration constant and depends on the initial conditions.

If we know the relation between π, x and t, we find the wave width Δx, i.e., the distance from the point at which at a given instant $\pi = 0$, to the point at which shock wave pressure π^* is attained

$$\Delta x = x(\pi^*) - x(0) = A\sqrt{\alpha + \frac{k+1}{2k} c_0 t}. \qquad \text{(XXIV-17)}$$

Thus Crussard could establish that in the one-dimensional case the amplitude of the shock wave drops with respect to its propagation, as $1/\sqrt{t}$, and the wave width increases proportionally with $\sqrt{t}$.[61] Crussard's original paper, written in 1912 - 1913, also contains an analysis which shows that this law applies to the case of small amplitude, that is an extreme law for a long propagation time.

In 1938, Shmushkevich [115] derived the same law in the following way. Assuming that the distribution of pressure in the wave remains similar with respect to propagation, (at least within the limit, with large t, after the wave has covered a long path), Shmushkevich writes the equation of the rate of wave width change Δx and compares it with the equation of wave momentum constancy

$$\frac{d\Delta x}{dt} = D - c_0 = \frac{k+1}{4k} c_0 \pi^*. \qquad \text{(XXIV-18)}$$

$$\Delta x \cdot \rho \cdot \bar{u} = \Delta x \cdot \pi^* \cdot \text{const} = \text{const}. \qquad \text{(XXIV-19)}$$

In setting up the second equation (the momentum equation) we use the linear relation between velocity and pressure known from acoustics, and also use the assumption according to which the distribution remains similar to itself, so that $\bar{u} = \text{const} \cdot \pi^*$. The two equations mentioned above can be readily solved

$$\frac{d\Delta x}{dt} = \frac{k+1}{4k} \cdot c_0 \cdot \pi^* = B \cdot \frac{1}{\Delta x}, \qquad \text{(XXIV-20)}$$

$$(\Delta x)^3 = 2Bt + \text{const},$$

$$\Delta x = B_1\sqrt{t + B_2}, \quad \pi^{*} = \frac{B_3}{\sqrt{t + B_2}}, \qquad \text{(XXIV-21)}$$

where B, B_1, B_2 and B_3 are constants.

Both Crussard and Shmushkevich assume that after the passage of the wave the substance returns to its initial state, with initial sound velocity c_0 and initial pressure $p = p_0$, $\pi = 0$. We disregard here the effects that depend on entropy change, which are proportional to the cube of the amplitude. This is permissible because we deal with equations that contain greater terms.

Instead of the change in wave width (Shmushkevich's investigations), we could also study the change of its free energy which depends on the conversion of energy into heat, i.e., on the increase in entropy proportional to π^{*3},

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}(\text{const} \cdot \pi^{*2} \cdot \Delta x) = -\text{const} \cdot \pi^{*3} \qquad \text{(XXIV-22)}$$

$$\text{const} \cdot \pi^{*} \cdot \Delta x = \text{const}, \qquad \text{(XXIV-19)}$$

whence we get

$$\frac{d\pi^{*}}{dt} = -B\pi^{*3}. \qquad \text{(XXIV-23)}$$

Integration of Eq. (XXIV-23) gives a result which is identical with Eq. (XXIV-21).

Thus, by using the various properties of a shock wave (the fact that velocity $D < c + u$ (Crussard), the fact that $D > c_0$ (Shmushkevich), and the increase of entropy in the wave) we obtain an identical extreme law. This result depends on the inner connection between the properties of the wave mentioned above (see Chapter 11).

The experimental study of one-dimensional propagation of a shock wave was performed by Vieille [86], and later by Vautier [123], whose experiments are briefly described in Chapter 15.

Considerably more complex is the problem of limit laws of the propagation of spherical waves (over great distances). This problem is particularly interesting for studying the theory of brisance of explosives.

We shall begin the study of spherical shock waves by going back to the analogy of spherical acoustic waves.

The fundamental property of the latter is the decrease in amplitude which is inversely proprotional to the distance from the symmetry pressure. This decrease is not connected with a decrease in the total reserve of acoustic energy. The decrease in amplitude depends on the fact that as a spherical wave propagates, the amount of substance involved in its motion increases proportionally to the volume of the spherical layer.

The second property of spherical waves consists in that a compression wave is necessarily followed by a rarefaction (expansion) wave. If at the initial instant the center was surrounded by a compressed substance (Fig. 61a), its expansion causes a compression wave which is followed by a rarefaction wave (Fig. 61b, ABC and CDE). We also have two regions where pressure increases (AB and DE) and one region where pressure drops (BCD).

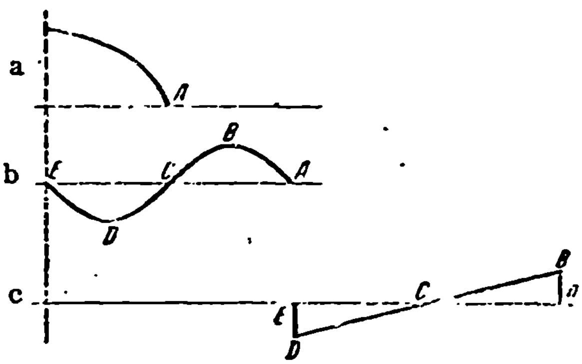

Fig. 61.

The dependence of propagation rate on amplitude causes a decrease in distances AB and DE, and an increase in distance BD. Landau [128] notes that at limit, after a sufficient amount time has elapsed (and after a sufficiently long path has been covered) the wave takes on a form that is shown in the bottom part of Fig. 61c with two shock waves AB and DE.

From the instant the shock wave has been formed, further propagation is accompanied by dissipation of acoustic energy, and by its conversion into thermal energy. The amplitude of maximum pressure drops faster than before, faster than according to the 1/r law.

Let us now find the quantitative rules, conserving the acoustic formula in Chapter 3

$$\pi = \frac{\mu(r - c_0 t)}{r} \qquad \text{(XXIV-24)}$$

as a zero approximation. In the next approximation, instead of c_0 we substitute propagation rate c + u which corresponds to a given state. We determine the change in distance between a pair of points, e. g., m and n, to which correspond specific values of μ_m and μ_n as the wave propagates

$$\frac{dr_{mn}}{dt} = (c+u)_m - (c+u)_n = c_0\left(1 + \frac{k+1}{2k}\frac{\mu_m}{r}\right) - c_0\left(1 + \frac{k+1}{2k}\frac{\mu_n}{r}\right) =$$

$$= c_0\frac{k+1}{2k}\frac{\mu_m - \mu_n}{r}, \qquad \text{(XXIV-25)}$$

$$\frac{dr_{mn}}{dr} = \frac{1}{c_0}\frac{dr_{mn}}{dt} = \frac{k+1}{2k}\frac{\mu_m - \mu_n}{r}, \qquad \text{(XXIV-26)}$$

$$r_{mn} = r_{mn0} + \frac{k+1}{2k}(\mu_m - \mu_n)\ln\frac{r}{r_0}. \qquad \text{(XXIV-27)}$$ [62]

We study the segment AB, and identify m = A, n = B, since $\mu_B > \mu_A$: Segment AB grows smaller with the motion of the wave. At a distance r such that

$$\ln\frac{r}{r_0} = \frac{r_{AB0}}{\mu_B - \mu_A}\cdot\frac{2k}{k+1}, \qquad \text{(XXIV-28)}$$

the length of segment r_{AB} becomes zero, i.e., a shock wave is formed. This also applies to DE.

Conversely, the length of segment BCD, on which pressure drops, increases as the wave propagates, so that the derivative $\partial\mu/\partial r$ decreases as t and r increase,

$$\frac{\partial\mu}{\partial r} = \frac{\mu_D - \mu_C}{r_{CD}} = \frac{1}{a + \frac{k+1}{2k}\ln r}, \qquad \text{(XXIV-29)}$$

where

$$a = \left(\frac{\partial \mu}{\partial r}\right)_0^{-1} - \frac{k+1}{2k} \ln r_0.$$

Let us now study the law governing the change in shock wave amplitude. The quantity μ^* in the shock wave front drops because the shock wave propagation velocity is smaller than the state propagation velocity with constant value of u. In analogy with the one-dimensional case we find

$$\frac{d\mu^*}{dt} = -(c+u-D)\frac{\partial \mu}{\partial r},$$

$$c_0 \frac{d\mu^*}{dr} = -\frac{k+1}{4k} c_0 \frac{\mu^*}{r} \cdot \frac{1}{a + \frac{k+1}{2k} \ln r}. \quad \text{(XXIV-30)}$$

This equation can be readily integrated

$$\mu^* = \frac{\text{const}}{\sqrt{a + \frac{k+1}{2k} \ln r}},$$

$$\pi^* = \frac{\mu^*}{r} = \frac{\text{const}}{r\sqrt{a + \frac{k+1}{2k} \ln r}}. \quad \text{(XXIV-31)}$$

As we compare this result with one-dimensional propagation, we find a curious formal analogy: the dependence of π on x in the one-dimensional case has the same form as the dependence of $\mu = \pi r$ on ln r in the spherical case.

Computations for the spherical case lead to the following conclusions:

1. The additional drop in amplitude, specific for shock waves, turns out to be very small at great distances, as $(\ln r)^{-1/2}$, as compared with the acoustic drop (r^{-1}).

2. The limit form of the wave to which it tends when $r \to \infty$, becomes determined when ln r becomes sufficiently great. This requirement is much more stringent than the one according to which r must be large. A high value of ln r can be attained for such large r for which the absolute value of the wave amplitude becomes so small that its propagation loses interest altogether. New factors may be involved in the case of long propagation time.

The application of limit laws requires therefore great care. More than in any other case one has to resort to experimental data despite their incompleteness.

Figure 59 showed the curves of the change of pressure with time measured at varying distances from the explosion site. These curves are taken from the paper of the well-known English physicist Bernal, "The Physics of Air Raids", published in 1941 [101]. At the right-hand side of this figure we also give the metric measurements. The transition from curves $\pi(t)$ for r = const to the instant propagation of pressure in space $\pi(r)$ for t = const is quite complex since the propagation velocity and the amplitude are not constant.

To give an approximate idea of the thickness of the layer involved at each single instant by the distubance, in addition to the time scale we also give the c_0t scale, which is the product of time by sound velocity c_0 in unperturbed air.

What can we learn from Fig. 59? Tests confirm the existence of an expansion wave which follows the compression wave. At great distances the product of mean amplitude times expansion wave width approaches an identical value of the compression wave. The force momentum acting over a wide time interval (0.015 - 0.020 sec, as can be seen from the drawing) represents the difference in the effect of the compression wave and that of the expansion wave running in an opposite direction. This is why the force momentum drops faster than the wave amplitude.

In the theoretical portion, following Landau, we established that the limit form of the wave is distinguished by two pressure discontinuities, one in front and one in the back (see Fig. 60c). Bernal's curves do not show the formation of a pressure discontinuity in the back. By the shape of the last part of the expansion wave we shall precalculate the the distance at which this discontinuity has to take place.

We choose a curve with a well-expressed expansion wave recorded at a distance of 20 feet from the charge (second from the top, Fig. 59).

For $r_0 = 6$ m minimum pressure amounts to $\pi_{min} = -0.04$, and the distance of the minimum pressure m from point n at which pressure is restored amounts to about $r_{mn0} = 3$ m,

$$\mu_m = -0.04 \cdot 6 = -0.24, \ \mu_n = 0.$$

$$r_{mn} = r_{mn0} + \frac{k+1}{2k}(\mu_m - \mu_n)\ln\frac{r}{r_0} = 3 - \frac{6}{7} \cdot 0.24 \ln\frac{r}{r_0} \cdot \qquad \text{(XXIV-32)}$$

Assuming that $r_{mn} = 0$, we find the distance r at which the discontinutiy is formed:

$$\ln\frac{r}{r_0} = \frac{3 \cdot 7}{0.24 \cdot 6} = 14.5; \quad r = r_0 \cdot e^{14.5} = 12 \cdot 10^6 \text{ м}.$$

The wave will take on its extreme shape at a distance of 12,000 km. It is obvious that in this case all the statements referring to extreme shape have no realistic importance. The calculation leads us to conclude that in the case of spherical propagation, the formation of a shock wave on account of the dependence of propagation velocity on amplitude occurs very slowly. The front shock wave, in which pressure increases with a jump up to maximum values, is not formed in this fashion. Rather, it is formed at the instant when the detonation of the charge is terminated and there occurs a contact of the explosion products with the surrounding air. At this instant (at a distance from the center equal to the charge radius, about 6 cm for the charge to which Fig. 59 refers) its amplitude attains enormous values (see Chapter 23). With further propagation the amplitude drops, but the increase in pressure maintains the character of a shock wave.

There exists an extensive literature on the subject of pressure amplitude in a shock wave following an explosion. Older data, however, must be used with great circumspection since for a correct measurement of a rapidly changing pressure sufficiently inertialess devices are required.

In most cases the surface of the device receiving the pressure was placed in a direction facing the propagation of the wave. When the wave reached the surface, it was reflected by it. Peak pressure increases two-fold in the case of weak waves, and even more in the case of great amplitudes (see Chapter 19). After processing the data of many authors, Vlasov derived the dependence

$$p_m = p_0 + 2.4\frac{\sqrt[3]{M}}{r} = p_0 + 44\frac{R}{r}, \qquad \text{(XXIV-33)}$$

where p_m is the pressure developed at the reflection of the blast wave,

p_0 is atmospheric pressure,

r is the distance from the explosion center, expressed in meters,

R is the effective charge radius (in m),

M is the weight of the charge (in kg); this formula holds true for explosives of the TNT types: other explosives, varying considerably in their power, require the introduction or corrections.

Vlasov limits the applicability of his formula by the condition $r > 85R$, $r > 4.4\sqrt[3]{M}$ (the dependence is stronger than Eq. (XXIV-33) in the case of smaller distances). For the entire interval investigated by him, Sadovskiy gives the following expression for maximum pressure

$$p_m = p_0 + 12\frac{\sqrt[3]{M}}{r} - 22\frac{\sqrt[3]{M^2}}{r^2} + 147\frac{M}{r^3}. \qquad \text{(XXIV-34)}$$

Thus, at great distances Sadovskiy's formula[63] gives pressure amplitude which is five times greater than the one yielded by Vlasov's formula: the coefficient for the highest term is 12 instead of 2.4. To clarify the real value of the amplitude we refer first of all to Bernal's data (Table 6).

The first four columns show the data from Bernal's experiments. According to maximum pressure p_1 measured by him (read from the diagrams), pressure p_m is computed according to the formula of shock wave reflection, Eq. (XIX-2). We see from the table that beginning with $p_1 - p_0 \leqslant 0.15$, virtually $p_m - p_0 = 2(p_1 - p_0)$. Great distances do confirm Vlasov's formula.

Table 6

r, фут a)	$\frac{r}{\sqrt[3]{Li}}$, $\frac{м}{\sqrt[3]{кг}}$ b)	Бернал c)		Власов d)	Садовский e)
		p_1-p_0	p_m-p_0	p_m-p_0	
10	3.05	1.8	6.9	—	[illegible]
20	6.10	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	0.16	[illegible]	[illegible]

CODE: a) Feet; b) kg; c) Bernal; d) Vlasov;
e) Sadovskiy.

Could it be that the high pressures recorded by Sadovskiy are of very short duration, hence they have not been recorded by other authors using other methods? The best way to verify this is to set up a comparison with the propagation velocity of a shock wave that depends on amplitude (Table 7).

Table 7

$r/\sqrt[3]{Li}$	a) Скорость волны		
	b) Садовский	c) Бернал	d) эксперимент [113]
4.3	[illegible]	470	—
8.6	414	[illegible]	[illegible]
12.9	[illegible]	[illegible]	[illegible]

CODE: a) Wave velocity; b) Sadovskiy; c) Bernal;
Experiment [113].

If we assign a specific value to the dependence of pressure on distance, we can find the values for velocity at any point. Computation of mean velocity requires a more complex procedure. In the table these values are compared with experimental data of French researchers taken from Savich [113], which determine the velocity of the wave. This comparison is also unfavorable for Eq. (XXIV-34).

Let us finally note that the assumption of a sharp pressure peak contradicts the theoretical concepts. Such a peak should be subjected to an exceedingly rapid weakening and expansion. From Bernal's curves we can find determine the rate of pressure change after shock compression and hence the law of amplitude change of the shock wave proper.

If for distances of 10 - 40 - 200 m (for a charge of 1 kg) we approximate the real law governing amplitude drop by the power function $\pi = \text{const} \cdot r^{-\nu}$, then the value of exponent ν within these limits drops from 1.4 to 1.25. At great distances the simple formula $\pi = \text{const} \cdot r^{-1}$ gives a satisfactory approximation to the true law.

We noted in Chapter 21 that the duration of the effect of blast wave pressure is proportional to the linear dimensions (e.g., the radius) of the charge. The magnitude of the time involved will be obtained by setting up the ratio of the charge radius to sound velocity R/c_0.

Bernal's data show that the action time of a compression wave τ amounts to from 0.03 to 0.05 sec, whereas R/c_0 for his charge amounts to 0.06/330 = 0.0002". Thus, the dimensionless ratio $\tau : \frac{R}{c_0}$ varies from 15 to 25 and thus differs noticeably from unity. The long duration and, consequently, the considerable expanse of the blast wave are quite natural. Wave width and duration of effect are maintained during propagation of a weak acoustic wave. We would have $\tau : \frac{R}{c_0} \approx 1$ in the case where the initial disturbance could be regarded as weak, i.e., if the change in pressure in the region taken by the explosive were small.

In reality, however, during the first stages of propagation the pressure amplitude is huge, hence the acoustic approximation is completely inapplicable. It can be regarded as approximately correct only from the instant when mean pressure in the region covered by the disturbance drops to 1 atmosphere. For conventional explosives the volume of this region reaches 10 m^3 per 1 kg, to which corresponds a radius $R' = 1.35 \sqrt[3]{M}$ (m, kg). The radius R' of the region in which mean pressure equals 1 atmosphere (2 atmospheres absolute) is 22 times greater than the charge radius. In accordance with our ideas the magnitude of $\tau : \frac{R'}{c_0}$ is actually of the order of unity.

Because of the great width and long duration of the wave, the momentum of the pressure acting on the body's surface normal to the wave depends to a great extent on the conditions of wave reflection and of the flow around the body of the air set in motion by the wave. Apparently this is why there are so many contradictions in the scientific and experimental literature on this subject.

Bernal's curves make it possible to find (even though with poor accuracy) the efficiency of the conversion of explosive energy into blast wave energy. Blast wave energy consists of the kinetic energy of air motion and potential energy (equal to the work performed by the change in air pressure). It is obvious that both compression and expansion of air under atmospheric pressure require an output of energy and increases the system's potential energy.

Total energy of a unit of volume is approximately equal to 25 $(\Delta p/p)^2$ kcal/m^3. Calculations for a distribution that corresponds to Bernal's curves yields an efficiency of about 30 - 40%. The energy of the compression wave and that of the expansion wave are at an approximate ratio of 3 : 1.

Thus, the energy of an explosive is converted into blast wave energy and is transferred over a distance exceeding hundred and thousand-fold the size of the charge, with an efficiency of the same order as the one for the conversion of gunpowder energy into motion energy of the projectile in the gun or combustion energy of the fuel into mechanical energy in the engine.

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FOOTNOTES

p. 7. [1]This equation refers to a specific combination of molecules of a fluid (Lagrange representation). According to Euler's representation for a specific volume fixed in space, the energy equation has a more complex form.

p. 7. [2]This equation is applied by us to a substance the state of which is fully determined by a specific volume v and specific entropy S. It is not applicable, for instance, to a system which is not in chemical equilibrium, in which during motion there occurs an irreversible chemical reaction.

p. 7. [3]The general gas dynamics equations that take account of viscosity and thermal conduction are given in the Appendix at the end of the present Chapter. The reader can skip this Appendix without impairing his understanding of what follows, if he takes for granted the statement regarding the applicability of Eqs. (I-1) - (I-6).

p. 18. [3a]We use the transformations

$$\frac{\partial}{\partial x} f(x-ct) = f'; \quad \frac{\partial}{\partial t} f(x-ct) = -cf'.$$

p. 21. [4]The flow of a substance through a spherical surface with radius r is $4\pi r^2 u$. The difference in the flows of substance that have crossed spheres with radii r and r + dr, is the amount of substance that remains in a spherical layer with a volume equal to $4\pi r^2 dr$, and it changes the density of the substance enclosed in that layer.

p. 24. [5]Over and above the amount contained in a given volume with a nonturbulent density value.

p. 28. [6]The relation $\left(\frac{\partial p}{\partial v}\right)_s = \frac{c_p}{c_v}\left(\frac{\partial p}{\partial v}\right)_T$ in its general form can be derived from the fundamental hydrodynamic relations for any system, and not only for an ideal gas in which c_p and c_v depend only on T (see Landau and Lifshits [15, p. 48, problem]. The direct measurement of $\left(\frac{\partial p}{\partial v}\right)_s$ or c_v is extremely difficult in the case of liquids. For the computation one uses the thermodynamic relation

$$c_p - c_v = -T\left(\frac{\partial p}{\partial T}\right)_v^2 \Big/ \left(\frac{\partial p}{\partial v}\right)_T$$

(ibid., problem No. 11), whence

$$\left(\frac{\partial p}{\partial v}\right)_s = \frac{c_p\left(\frac{\partial p}{\partial v}\right)_T^2}{c_p\left(\frac{\partial p}{\partial v}\right)_T + T\left(\frac{\partial p}{\partial T}\right)_v^2}.$$

Finally, the quantity $\left(\frac{\partial p}{\partial T}\right)_v$ can be expressed by means of isothermic compressibility and the coefficient of thermal expansion—a relation common to any three quantities connected by one equation—by the equation of state p = p(v, T) in the given case

$$\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial T}{\partial v}\right)_p\left(\frac{\partial v}{\partial p}\right)_T = -1.$$

(Max Planck, Thermodynamics, Chap. 1), so that

$$\left(\frac{dp}{\partial T}\right)_v = -\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_p.$$

The connection between the derivatives with respect to density in (II-30, II-31) and the derivatives with respect to volume is elementary

$$\varrho = \frac{1}{v}; \quad \frac{\partial p}{\partial \varrho} = -v^2\frac{\partial p}{\partial v}.$$

p. 29. [7]Later measurements by Wallmann [126] yielded a second, smaller number of collisions.

p. 30. [8]To return precisely to point A, this heat should be marked on BC or A'A. However, the heat sampled during the cycle and, accordingly, the shift of the initial point in the case of absence of heat sampling, are of a smaller order of magnitude than shifts AB, AA', AC and BC in Fig. 3. We have disregarded them in the text and in Fig. 3.

p. 35. [9]The beginning of the process—the change of the form of wave b—is taken as a change in the spectral composition of sound, as the appearance of overtones (which can be proved by expanding curve b in a Fourier series) and the change in tone when sound propagates over great distances (see Thuras, Jenkins & O'Neil [94, 52, 53] and also a similar paper by Eykhenval'd [34]).

p. 36. [10]See Landau and Lifshits [15], pp. 41-42, Chap. 13 "Steady Flow".

p. 41. [11]The history of the problem is brilliantly expounded in Stodola's manual [89].

p. 42. [12]The process of mixing and slowing down a jet was investigated by G.N. Abramovich (TsAGI) and S.N. Syrkin and Lyakhovskiy (TsKTI).

p. 43. [13]To obtain a satisfactory thermal efficiency in his steam turbine, Laval had to operate with a very wide pressure differential $p_0 - p_n$ that exceeded the critical one. In order to use it without losses, the switch to supersonic speed became necessary.

p. 47. [14]We can see from the formulas of Chapter 2 that in an incompressible liquid $\frac{\partial \varrho}{\partial p} = 0$, $\frac{dp}{d\varrho} = \infty$, $c = \infty$, the speed of sound is infinite, motions remains "subsonic" for any speed.

p. 47. [15]Figure 12 had been done at a reduced scale.

p. 49. [16]To write Eq. (IV-1) we use (III-5) and (III-16).

p. 51. [17]We will see in Chapter 17 that in the presence of a shock wave pressure is not entirely restored; the temperature, however, is completely restored up to the magnitude of "temperature at rest" in the case of deceleration.

p. 52. [18]We investigate heat transfer of the plate only with the gas. Heat transfer into the plate or radiation from the plate's surface reduce surface temperature (see Kibel's [10]).

p. 59. [19]The letters AB in Fig. 16a, b are totally unrelated to points A and B in Fig. 14.

p. 62. [20]The velocity tangential to surfaces A and B must be maintained in terms of magnitude and direction when the substance passes through the wave. Consequently, a tangential motion can be totally excluded from the investigation by a corresponding choice of a uniformly moving system of coordinates.

p. 77. [22]The constant addend that appears in I if the thermal capacity below T_1 differs from the thermal capacity in the interval from T_2 to T_1 contained in the formulas, can be eliminated by choosing correspondingly the energy reading point. In any event, the constant addent disappears from equations of the form (VIII-5) and (VIII-6).

p. 98. [23]H_A and H_B are the accepted abbreviations for Hugoniot's adiabatic curves, for which the subscripts A and B denote the initial point.

p. 100. [24]Eq. (IX-3) can be derived from Eq. (VIII-6) if from density we switch to specific volume.

p. 100. [25] T in Eq. (XI-5) is enclosed between T_C and T_B. To prove this we pass from state A to B (Fig. 29) by isentropic compression (AC) and subsequent heating of the compressed gas in a constant volume (CB)

p. 101. [26] We note that v_2 is smaller than v_1 so that $\omega < 0$.

p. 102. [27] It may be useful to point out another time that the calculation of the area of the trapezium limited by straight line AFB (Fig. 29) is based on the expression of Hugoniot's adiabatic curve which follows from the conservation laws applied to the state before and after the passage of the wave. This calculation is not connected in any way with the problem of the shape of the line along which in actual fact the state in the wave changes (see Chapter 12).

p. 102. [28] An incredibily rapid increase in thermal capacity is required for the absolute quantity $\left(\frac{\partial p}{\partial v}\right)_s = -k\frac{p}{v}$ to drop with increasing temperature on account of a drop in $k = c_p/c_v$.

p. 103. [29] The change in the quantity $\left(\frac{\partial p}{\partial v}\right)_s$, on which depends sound velocity when changing from A to C or from A to B, is of the first order in $v_1 - v_2$. The change in $\left(\frac{\partial p}{\partial v}\right)_s$ when passing from C to B is of the third order.

p. 103. [30] D, c_1 and c_2 with small amplitude differ by a quantity proportional to the amplitude. Velocity u is also proportional to the amplitude. With an accuracy up to quantities proportional to the square of the amplitude, shock wave velocity is equal to the arithmetic mean of sound velocity at initial state c_1 and disturbance propagation velocity in the direction of the wave in a compressed, moving gas $c_2 + u$

$$D = \frac{c_1 + c_2 + u}{2}.$$

p. 108. [31] In Fig. 32, Poisson's adiabatic curves passing through points A, B, and M are denoted by P_A, P_B and P_M.

p. 111. [32] In states A and B, obviously $du/dx = 0$. When integrating it must be borne in mind that $\rho u = \text{const}$ according to the equation of conservation of matter.

p. 113. [33] In all computations referred to above we took an ideal gas for which (at least in order of magnitude) there take place the following estimates

$$\left(\frac{\partial p}{\partial v}\right)_s \simeq -\frac{p}{v}; \quad \frac{\partial^2 p}{\partial v^2} \simeq \frac{p}{v^2}.$$

In the general case we can readily establish that, all other conditions being equal, the width of the front is inversely proportional to $\left(\frac{\partial^2 p}{\partial v^2}\right)_s$, depending on the role played by this quantity in shock wave theory.

p. 119. [34]We confine ourselves here to referring to Kan'yar who investigated motion with a small amplitude. Unlike other authors, he studied from the beginning those equations of motion that contain terms expressing viscosity so that his calculations cover not only the formation of shock waves, but also the steady-state structure of the wave front. There is little physical interest in such a study since the effect of viscosity prior to the formation of a shock wave is negligible, and the steady-state structure is found easier by direct methods which proceed from the assumption of a stationary wave.

p. 123. [35]All computations are referred to the state prior to the occurrence of the shock wave $t < t_b$, i.e., $t'<0$. Motion occurs in the region $x < x_b$, where $x' < 0$.

p. 125. [36]For $t \to t_b$ Eqs. (XIV-20) - (XIV-21) lead to $x_n \to x_b$, which corresponds to infinite compression (a finite amount of substance from segment $0 - x_b$ is compressed into an interval between x_n and x_b that approaches zero), infinite pressure and velocity.

p. 130. [37]Far away from resonance, Δp and w change with an appreciable phase shift, hence Eq. (XV-2) would be incorrect (too high).

p. 137. [38]For a diatomic gas with $c_p/c_v = 1.4$. In the general case, one will need for this a velocity in excess of $\frac{2c^*_A}{K_A-1} + \frac{2c^*_B}{K_B-1}$ where K_A and K_B are the adiabatic exponents of gases A and B.

p. 140. [39]c_0 is sound velocity in the air. Sound velocity in hydrogen is equal to $4c_0$.

p. 141. [40]A and B are not shown in Fig. 36, but they are used below in Fig. 41c. See also Figs. 39 and 40b.

p. 147. [41]$dI = vdp + TdS$; for $I = \text{const.}\ \frac{dp}{dS} = -\frac{T}{v}$.

p. 148. [42]Eq. (III-5) is true only for that system of coordinates in which the body and the shock wave rest. In the system of coordinates in which the unperturbed gas rests while the body moves, as the body comes closer the gas particles are subject to compression (gas enthalpy increases) and start moving. They also acquire a kinetic energy so that the sum $I + u^2/2$ increases. Eq. (III-5) cannot be applied in this system of coordinates.

p. 160. [43]In a jet-propelled missile the gunpowder burns under constant pressure, and develops a temperature that is lower than during combustion in a sealed container. Hence the power of gunpowder f, contained in Eqs. (XVIII-13) - (XVIII-14) must be reduced with respect to thermal capacity, i.e., by K = 1.25 times as compared with the power of the same gunpowder measured in a sealed container.

p. 165 [44]According to a remark by Landau, the abrupt increase in pressure in a shock wave causes simultaneously the separation of the boundary layer.

p. 168. [45]Belyayev defended his thesis in 1935. Similar calculations were performed independently by Vlasov [3].

p. 172. [46]Here we do not investigate the case when the reacting substance is enclosed in a hermetically sealed shell. Under such conditions, even a slow chemical reaction accompanied by liberation of gas, develops a very high pressure contained by the solid vessel. The rupture of a high-pressure vessel recalls an explosion in many ways, but the details of this process which depend on the properties of the material of the container, and on its design, do not interest us.

p. 172. [47]This velocity is different from the rate of the chemical reaction of a specific particle of the substance characterized by reaction time. As in the case of the propagation of the reaction, we must distinguish between reaction time of the entire charge (which, in the simplest case of constant velocity, is proportional to the size of the charge) and reaction time of individual particles of substances; reaction time of individual particles obviously represents only a portion of the former, since in an explosion the various particles do not react simultaneously, and in the afore-mentioned simplest case do not depend on the size of the charge.

p. 173. [48]In the case of a nonsymmetric propagation of the detonation, the distance covered by the explosion products and the power of the explosion is greater in some directions (mainly in the direction of detonation wave propagation) and smaller in others. Here we will not touch upon the extremely interesting and important problem regarding cumulative charges characterized by an extremely powerful concentration of energy in an assigned direction. This problem is studied by specialized literature [110].

p. 174. [49]The heat of TNT combustion in a calorimetric bomb with excess oxygen amounts to 3592 kcal/kg (with formation of water); combustion with formation of water vapor yields about 3480 kcal/kg. The heat of a TNT explosion with a high-density charge, according to Schmidt, equals 1085 kcal/kg. We find the heat of explosion products combustion by subtracting the explosion heat (3480 - 1085 = 2395) from TNT combustion heat.

p. 174. [50]The phenomenon of the barrel flame is well known. After the projectile has left the barrel, the gunpowder combustion products flow out and mix with the surrounding air. If they contain a sufficient amount of combustible and if the temperature is sufficiently high, the mixture burns up (explodes) with an intense flare.

In connection with the location of guns by the sound ranging method, Esclangon [114], followed by other authors, investigated the sound of a gunshot and discovered the existence of two separate sound waves: one produced by the expansion of the gunpowder combustion products, and another one produced by the barrel flames. At a great distance from the guns, the latter is more intense than the former and has a long wave length.

p. 175. [51]However, in this case it has to borne in mind that the magnitude of destruction, if it occurs, depends on the size of the charge (which determines the length of the action exerted by pressure). The independence of the presence or the absence of destruction from the duration of the effect exerted by pressure, as can be seen from what was said above, takes place even with a specific minimal reaction time, i.e., with a specific minimal charge.

p. 176. [52]The wave pressure momentum is denoted as $\int (p - p_0)dt$ where p_0 is atmospheric pressure (a constant quantity), $p = p(t)$ is pressure at the point under study at the passage of a shock wave which is unperturbed by obstacles or measuring devices.

p. 181. [53]If r is expressed in meters, and m in kilograms, then for a spherical charge of density 1.6, the value $\frac{r}{\sqrt[3]{m}} = 1$ corresponds to the ratio r/R = 19. For a point lying on the surface of the charge, computed in technical units $\frac{r}{\sqrt[3]{m}} = 0.053$.

p. 182. [54]The similarity is not exact since in explosion products with initial density there occur great deviations from the equation of state for an ideal gas, which depend on density. The proper volume of the molecules in explosion products gives a characteristic density. One can assume, however, that this circumstance is of no significance at the moment when the shock wave has travelled to reach a considerable distance from the charge, and the explosion products have expanded considerably.

p. 197. [55]See footnote 60.

p. 197. [56]In the case of an explosion of heavy metal compounds (lead azide, mercury fulminate) the high molecular weight of explosion products additionally increases density.

p. 198. [57]Temperature at rest of explosion products turns out to be higher than the initial temperature of explosion products (detonation temperature). This is characteristic for an unsteady expansion wave in which energy is being redistributed: kinetic energy of explosion products rushing ahead is generated in part from potential energy (expansion) in deeper layers. These relationships are shown in Fig. 20 where we can compare the relation between velocity (which determines kinetic energy) and pressure (which determines potential energy) for a steady flow in the nozzle for which the sum of enthalpy and kinetic energy is constant, and for unsteady expansion.

p. 199. [58]Vlasov's paper reflects incorrect views regarding the possibility of intermittent expansion waves. Fortunately this error has no practical effect on the numerical results. He also ignores the remark by Landau [1-7, 108] regarding the form of the equation of state.

Correction note: Computations by Landau and Stanyukovich [108] give for TNT a velocity of explosion products and air of 7800 m/sec, a shock wave pressure of 750 kg/cm^2 and an explosion product temperature of 1200°K.

p. 206. [59]More precisely to the state which differs from the initial one only by the quantities proportional, in the case of small amplitude, to the cube of the amplitude because of a change in entropy from compression in the wave.

p. 208. [60]We can readily see that initial distribution with constant sign $dp/dx > 0$ monotonically approaches linear distribution in time, since the linear term in pressure proportional to $c_0 \pi t$ increases.

p. 209. [61]We have simplified the relationship by taking $\frac{k+1}{2k} c_0 t$ to be greater than α or appropriately changing the instant when the time count begins.

p. 212. [62]In deriving Eq. (XXIV-27), in Eq. (XXIV-25) we assumed a simple relationship between c + u and π, and ignored the terms $\sim r^{-2}$. In Eq. (XXIV-26) we substituted c + u for c_0, assuming the amplitude to be small.

p. 216. [63]Proofer's remark: The formula was communicated by M.A. Sadovskiy in a paper in 1942. He found later [127] that all the factors have to be decreased by a factor of 1.92.